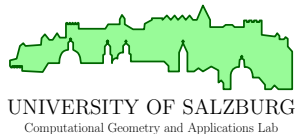


# Computation and Recognition of Weighted Skeletal Structures in the Plane

## Dissertation Defense

Günther Eder



- Min-/Max-Volume Roofs of Polygonal Footprints of Buildings
- Straight Skeleton of an Orthogonal Monotone Polygon
- Positively Weighted Straight Skeletons of Simple Polygons
- Recognizing Geometric Trees as Weighted Straight Skeletons
- Multiplicatively Weighted Voronoi Diagrams in the Maximum Norm

- Min-/Max-Volume Roofs of Polygonal Footprints of Buildings
- Straight Skeleton of an Orthogonal Monotone Polygon
- Positively Weighted Straight Skeletons of Simple Polygons
- Recognizing Geometric Trees as Weighted Straight Skeletons
- Multiplicatively Weighted Voronoi Diagrams in the Maximum Norm

- Min-/Max-Volume **Roofs** of Polygonal Footprints of Buildings
- **Straight Skeleton** of an Orthogonal Monotone Polygon
- Positively Weighted **Straight Skeletons** of Simple Polygons
- Recognizing Geometric Trees as Weighted **Straight Skeletons**
- Multiplicatively Weighted Voronoi Diagrams in the Maximum Norm

- Min-/Max-Volume **Roofs** of Polygonal Footprints of Buildings
- **Straight Skeleton** of an Orthogonal Monotone Polygon
- Positively Weighted **Straight Skeletons** of Simple Polygons
- Recognizing Geometric Trees as Weighted **Straight Skeletons**
- Multiplicatively Weighted **Voronoi Diagrams** in the Maximum Norm

- Min-/Max-Volume Roofs of Polygonal Footprints of Buildings

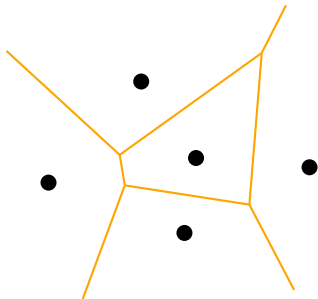
- Straight Skeleton of an Orthogonal Monotone Polygon

- Positively Weighted Straight Skeletons of Simple Polygons

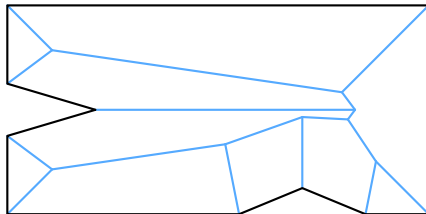
- Recognizing Geometric Trees as Weighted Straight Skeletons

- Multiplicatively Weighted Voronoi Diagrams in the Maximum Norm

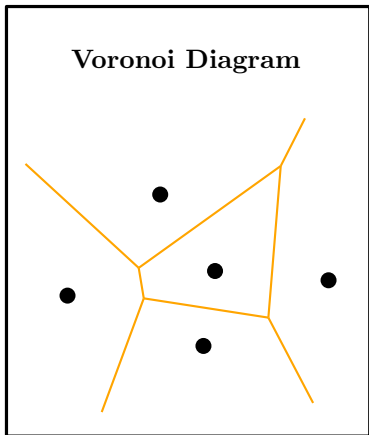
## Voronoi Diagram



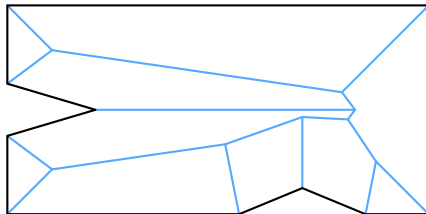
## Straight Skeleton



## Voronoi Diagram

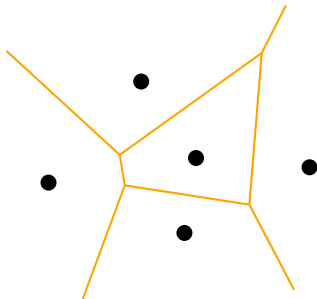


## Straight Skeleton



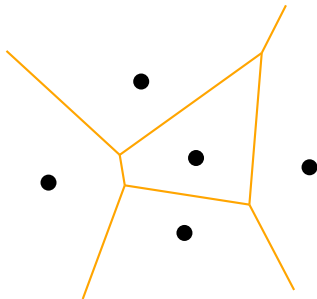


## Voronoi Diagram



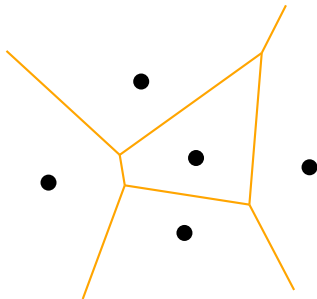
- Definition

## Voronoi Diagram



- Definition
- Applications

## Voronoi Diagram



- Definition
- Applications
- Our Result

# Voronoi Diagram – Definition

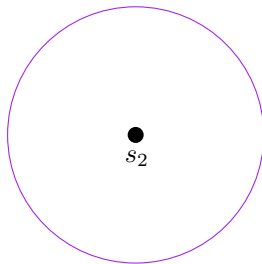
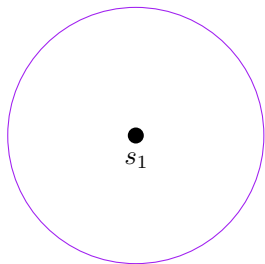
●  
 $s_1$

●  
 $s_2$

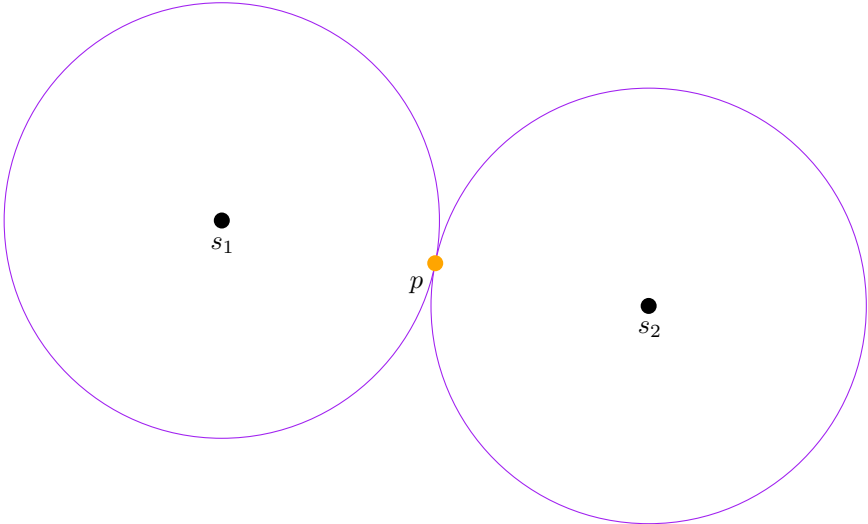
# Voronoi Diagram – Definition



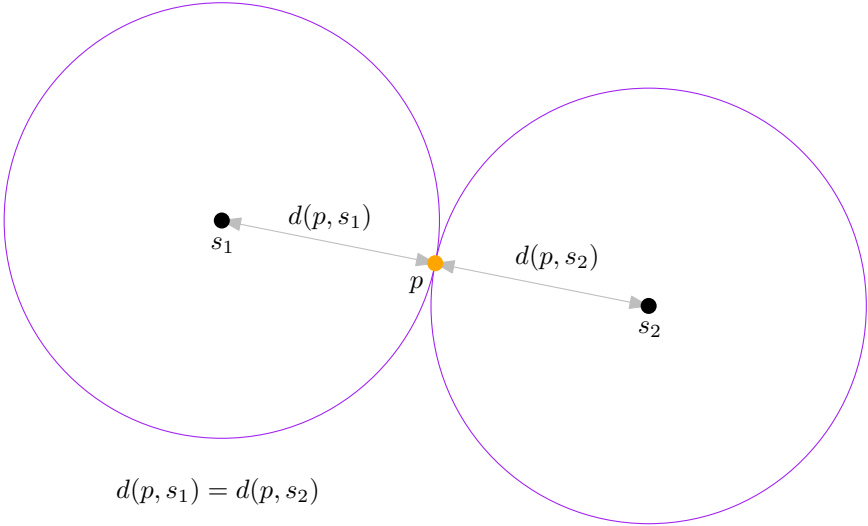
# Voronoi Diagram – Definition



# Voronoi Diagram – Definition

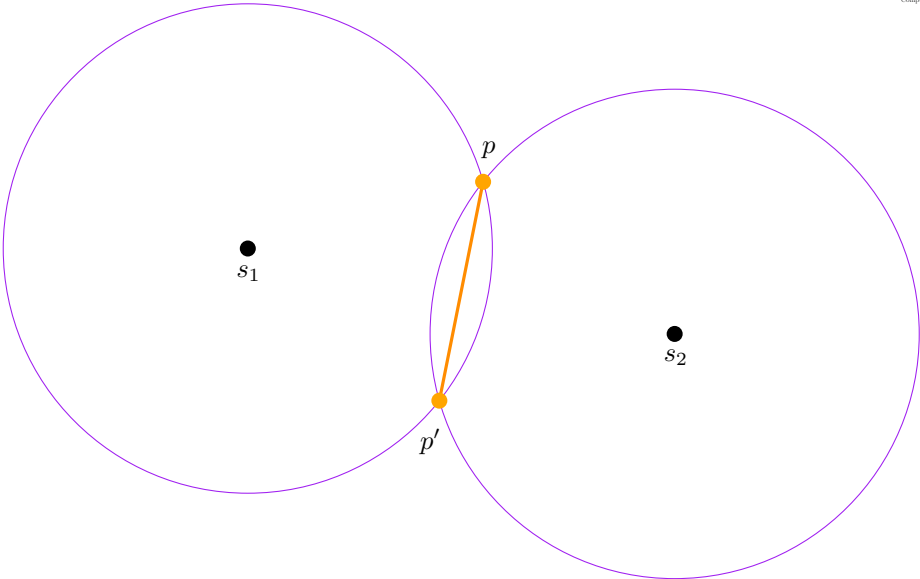


# Voronoi Diagram – Definition

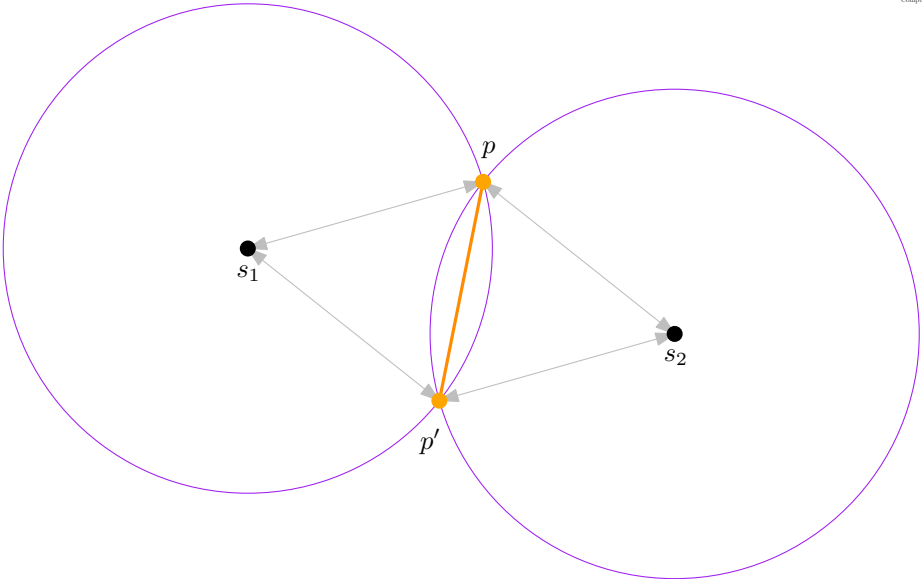




# Voronoi Diagram – Definition



# Voronoi Diagram – Definition



# Voronoi Diagram – Definition



$s_1$



$s_2$



# Voronoi Diagram – Definition



$s_1$



$s_4$

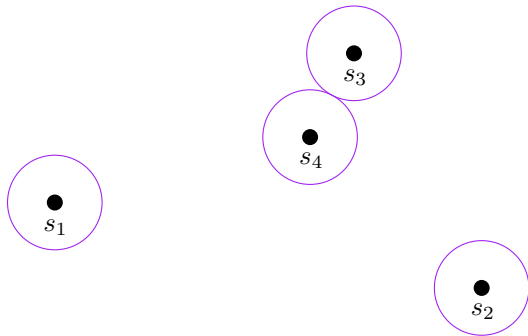


$s_3$

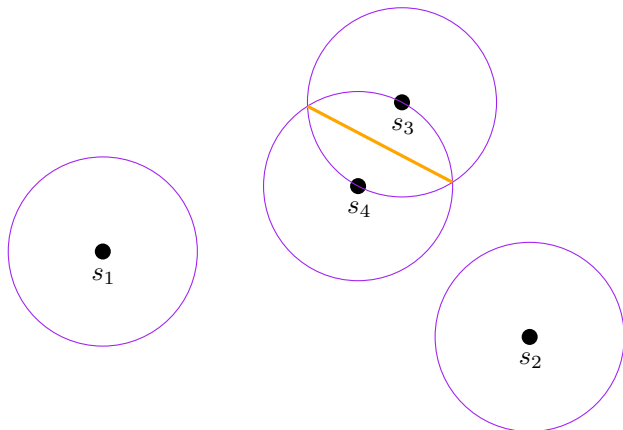


$s_2$

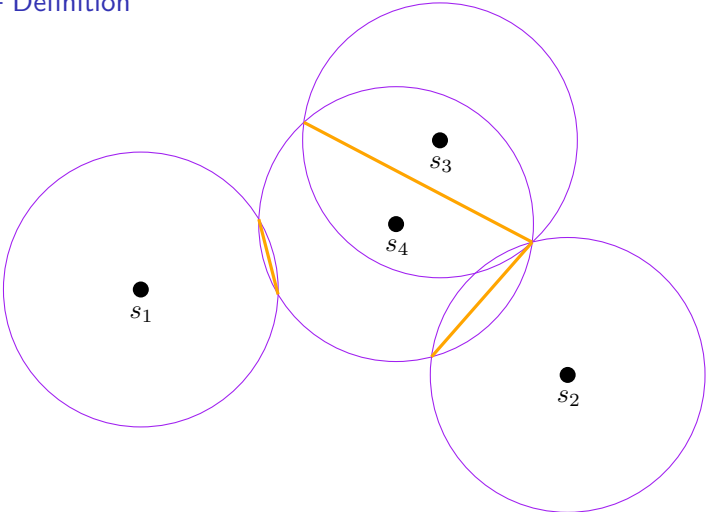
# Voronoi Diagram – Definition



# Voronoi Diagram – Definition



# Voronoi Diagram – Definition

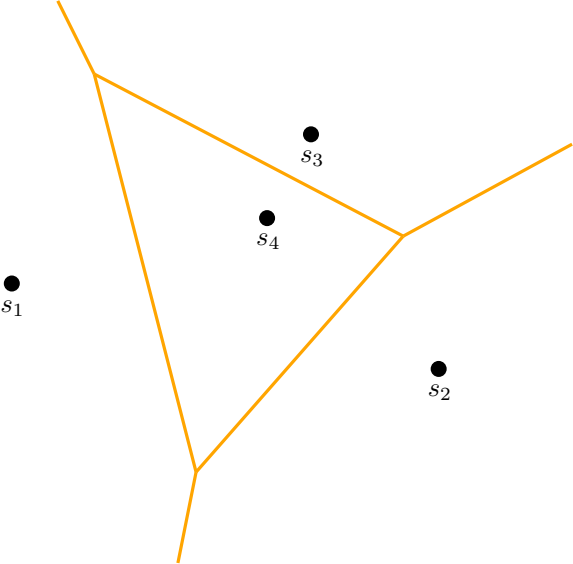


# Voronoi Diagram – Definition

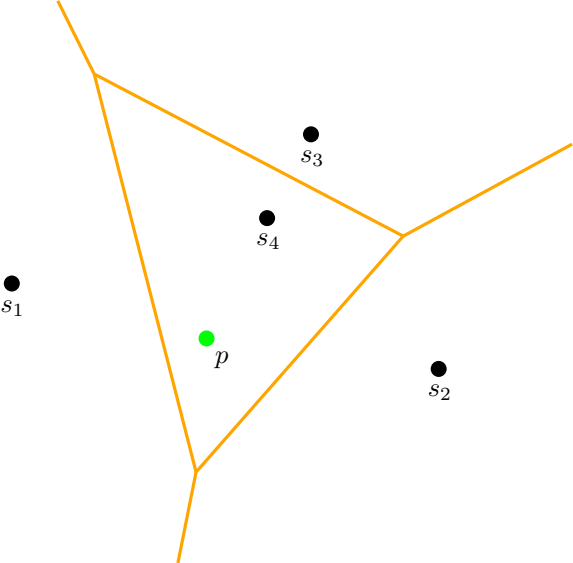




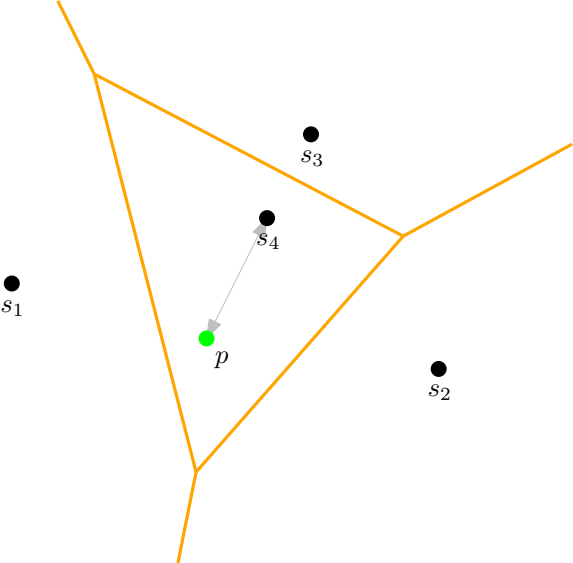
# Voronoi Diagram – Definition



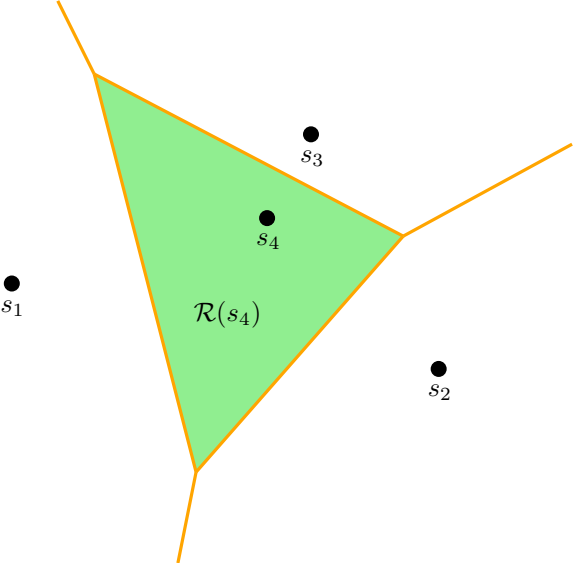
# Voronoi Diagram – Definition

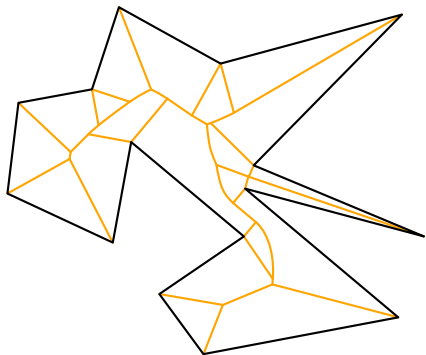


# Voronoi Diagram – Definition

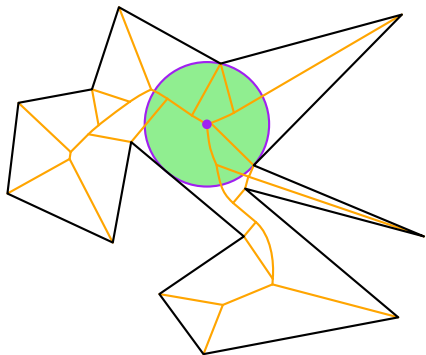


# Voronoi Diagram – Definition

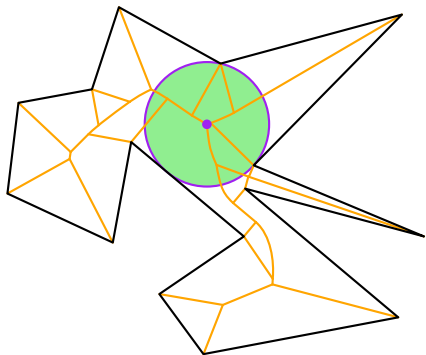




Maximum-Empty-Circle



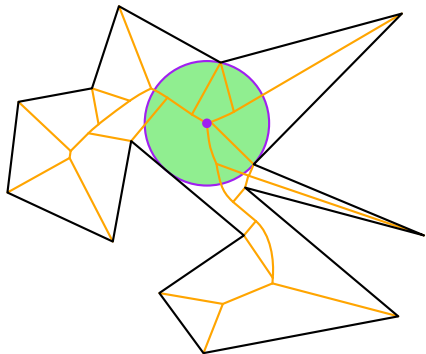
Maximum-Empty-Circle



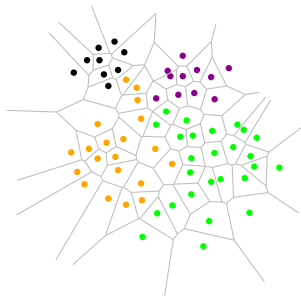
Maximum-Empty-Circle



1-NN Classification

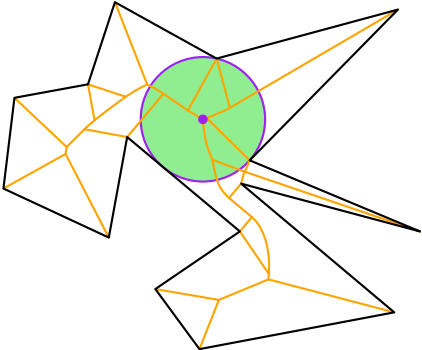


Maximum-Empty-Circle

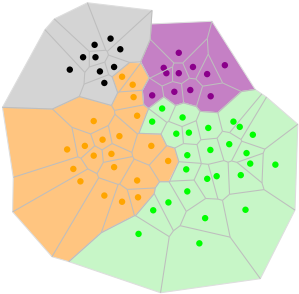


1-NN Classification

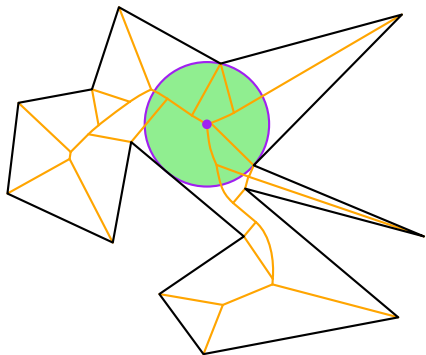




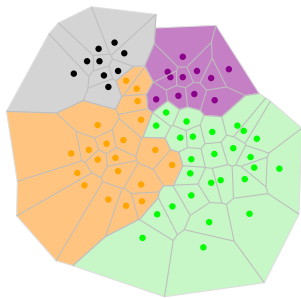
Maximum-Empty-Circle



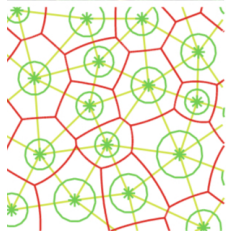
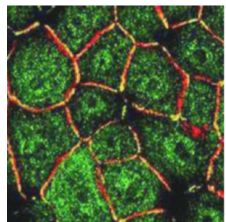
1-NN Classification



Maximum-Empty-Circle

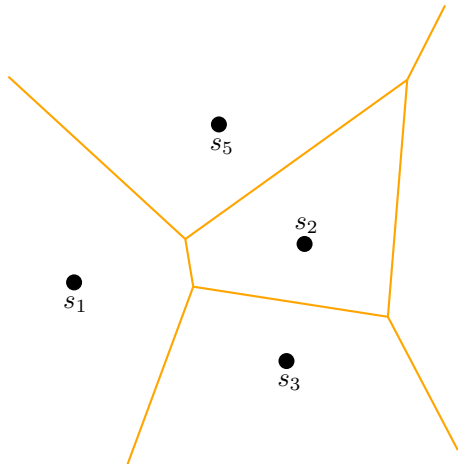


1-NN Classification

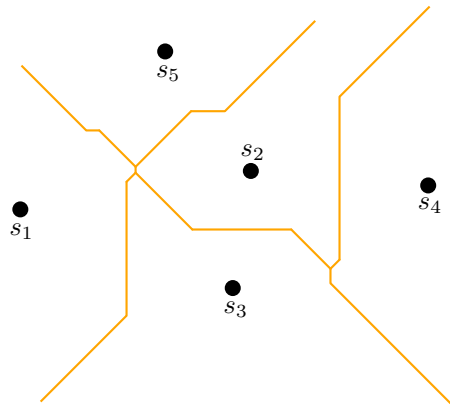


Cell Tissue Model  
from Bock et al.[1]

# Voronoi Diagram – $L_\infty$ & Weights

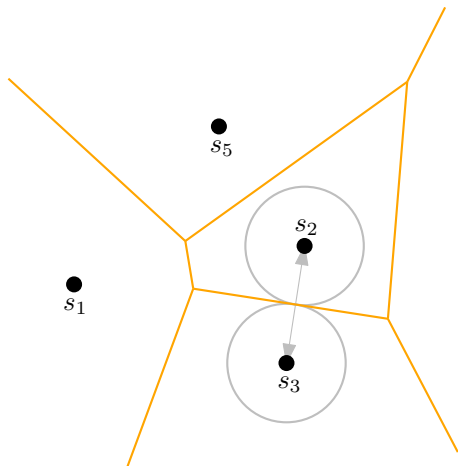


$L_2$ -Norm

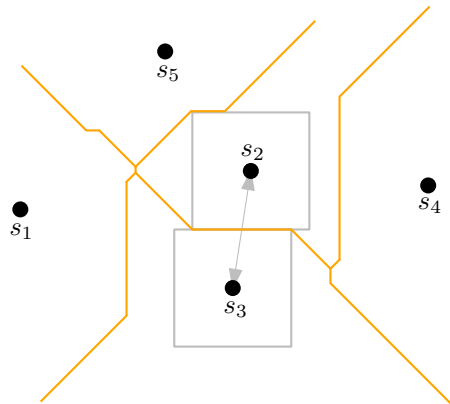


$L_\infty$ -Norm

# Voronoi Diagram – $L_\infty$ & Weights

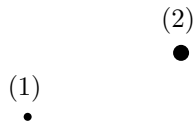


$L_2$ -Norm

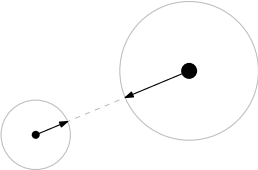


$L_\infty$ -Norm

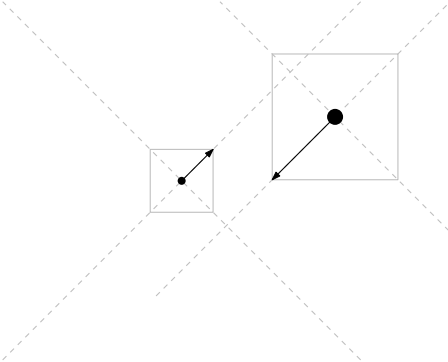
# Voronoi Diagram – $L_\infty$ & Weights



# Voronoi Diagram – $L_\infty$ & Weights

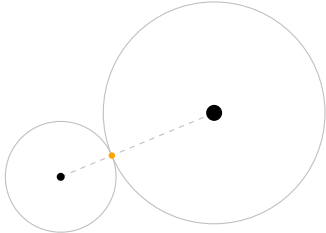


$L_2$ -Norm

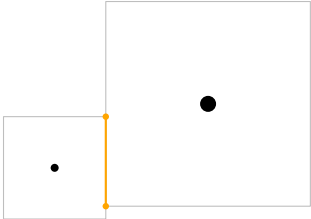


$L_\infty$ -Norm

# Voronoi Diagram – $L_\infty$ & Weights

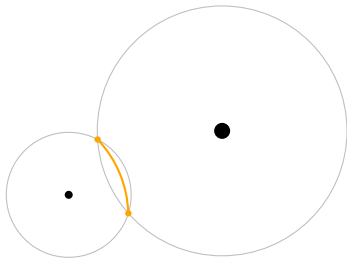


$L_2$ -Norm

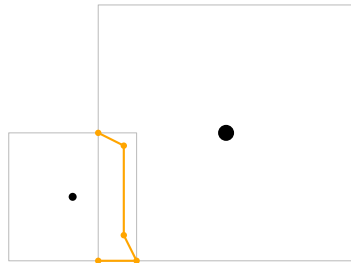


$L_\infty$ -Norm

# Voronoi Diagram – $L_\infty$ & Weights



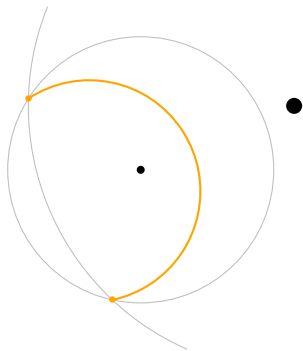
$L_2$ -Norm



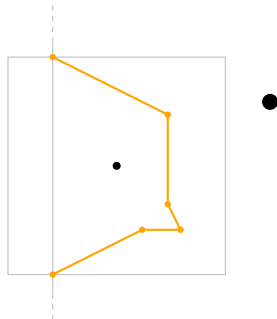
$L_\infty$ -Norm



# Voronoi Diagram – $L_\infty$ & Weights

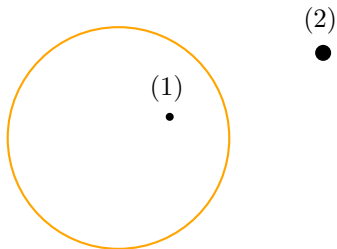


$L_2$ -Norm

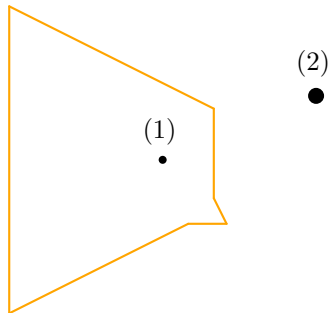


$L_\infty$ -Norm

# Voronoi Diagram – $L_\infty$ & Weights



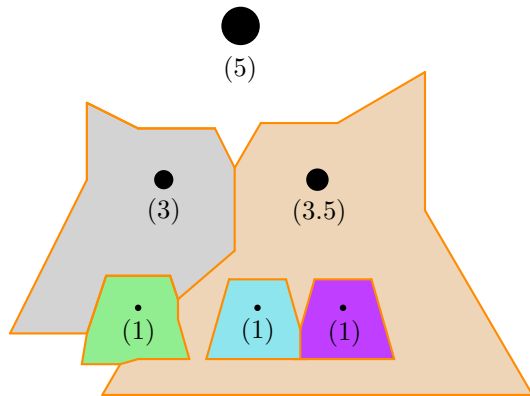
$L_2$ -Norm



$L_\infty$ -Norm

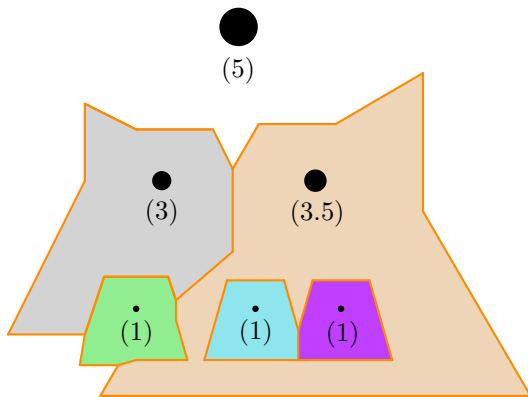
# Voronoi Diagram – Our Result

- Weighted Voronoi diagram in  $L_\infty$ -norm.
  - Points, line-segments, and rectangles.
- $\Theta(n^2)$  combinatorial complexity bound.
- $\mathcal{O}(n^2 \log n)$  incremental construction algorithm.
  - Compute a single region.
  - Embed region in diagram of previous step.



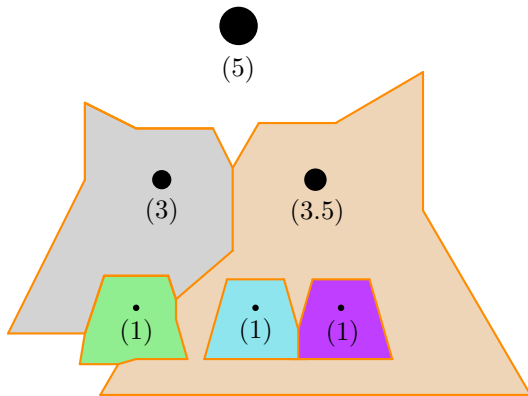
# Voronoi Diagram – Our Result

- Weighted Voronoi diagram in  $L_\infty$ -norm.
  - Points, line-segments, and rectangles.
- $\Theta(n^2)$  combinatorial complexity bound.
- $\mathcal{O}(n^2 \log n)$  incremental construction algorithm.
  - Compute a single region.
  - Embed region in diagram of previous step.

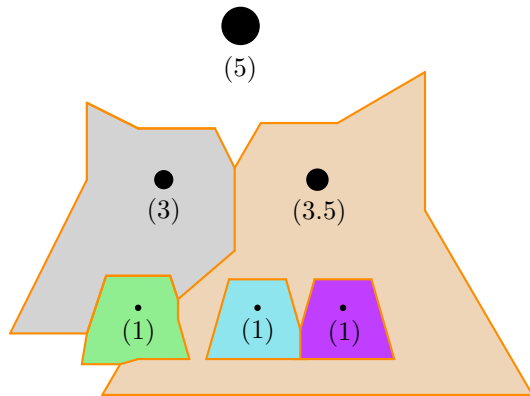


# Voronoi Diagram – Our Result

- Weighted Voronoi diagram in  $L_\infty$ -norm.
  - Points, line-segments, and rectangles.
- $\Theta(n^2)$  combinatorial complexity bound.
- $\mathcal{O}(n^2 \log n)$  incremental construction algorithm.
  - Compute a single region.
  - Embed region in diagram of previous step.

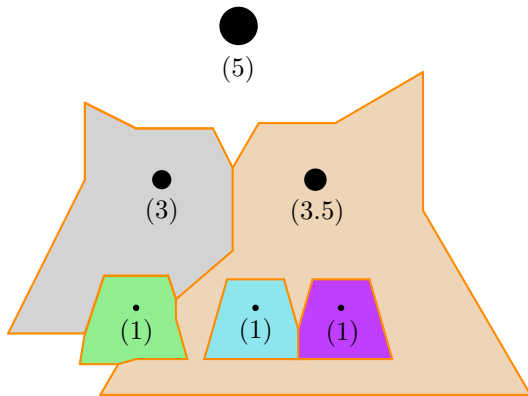


- Weighted Voronoi diagram in  $L_\infty$ -norm.
  - Points, line-segments, and rectangles.
- $\Theta(n^2)$  combinatorial complexity bound.
- $\mathcal{O}(n^2 \log n)$  incremental construction algorithm.
  - Compute a single region.
  - Embed region in diagram of previous step.



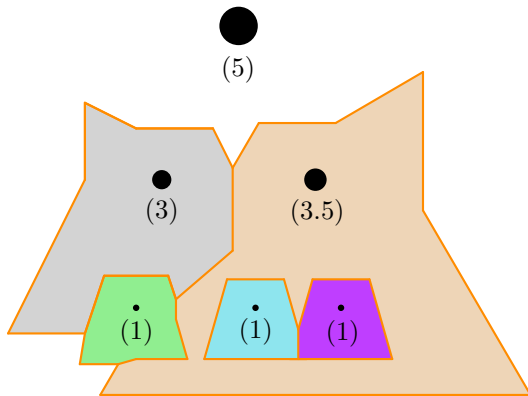
# Voronoi Diagram – Our Result

- Weighted Voronoi diagram in  $L_\infty$ -norm.
  - Points, line-segments, and rectangles.
- $\Theta(n^2)$  combinatorial complexity bound.
- $\mathcal{O}(n^2 \log n)$  incremental construction algorithm.
  - Compute a single region.
  - Embed region in diagram of previous step.



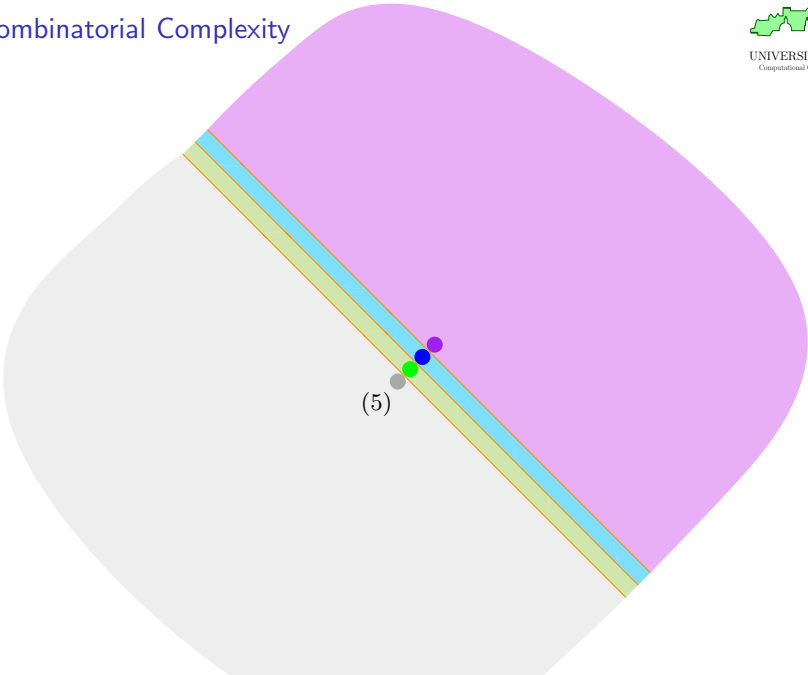
# Voronoi Diagram – Our Result

- Weighted Voronoi diagram in  $L_\infty$ -norm.
  - Points, line-segments, and rectangles.
- $\Theta(n^2)$  combinatorial complexity bound.
- $\mathcal{O}(n^2 \log n)$  incremental construction algorithm.
  - Compute a single region.
  - Embed region in diagram of previous step.

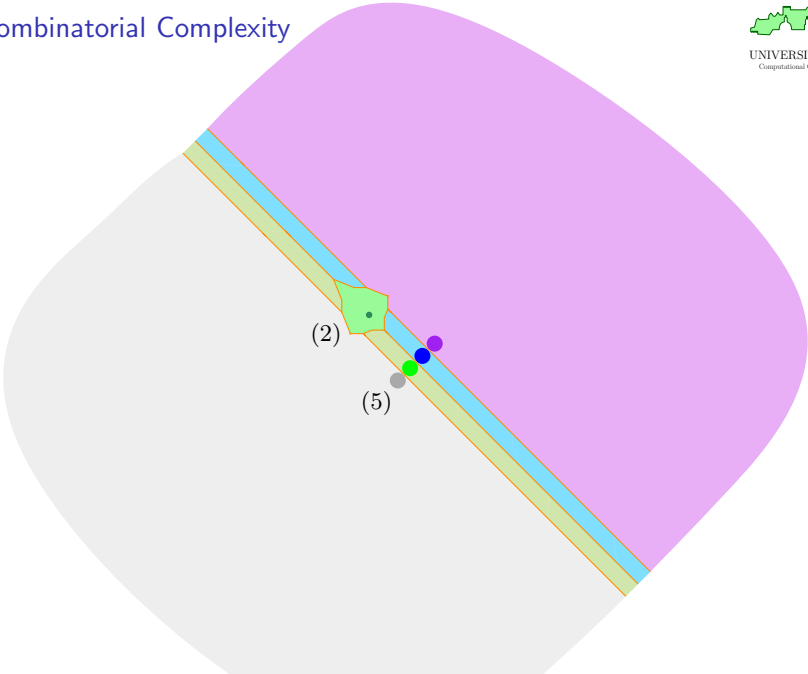




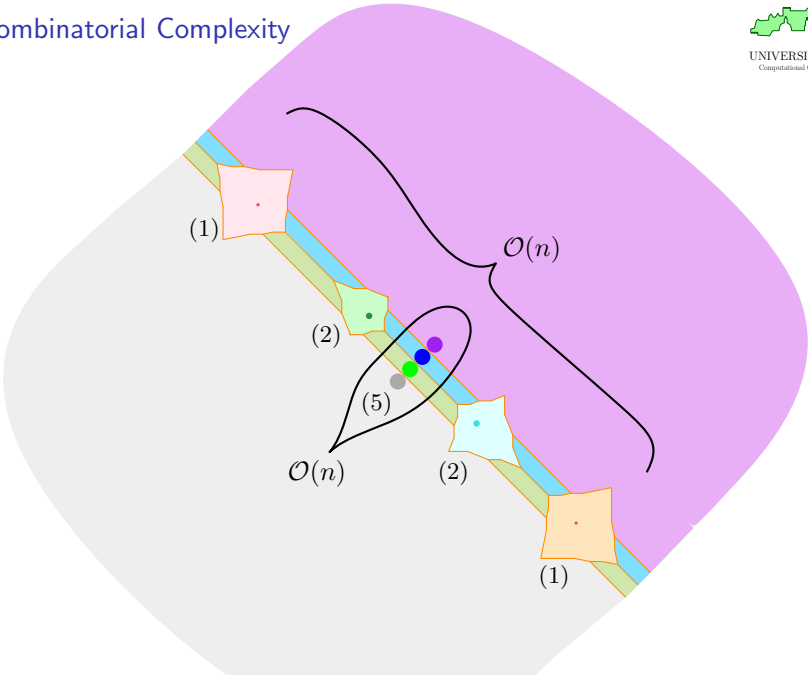
# Voronoi Diagram – Combinatorial Complexity



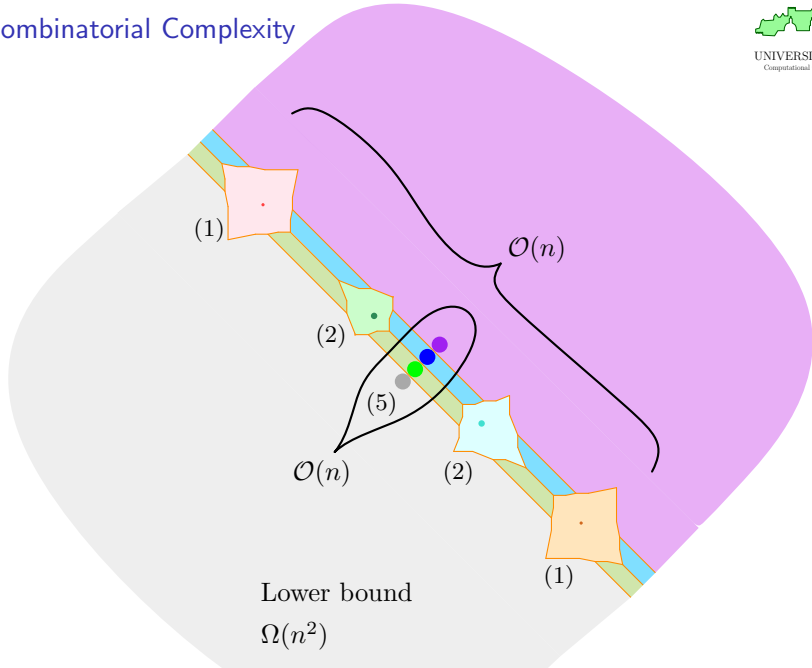
# Voronoi Diagram – Combinatorial Complexity



# Voronoi Diagram – Combinatorial Complexity



# Voronoi Diagram – Combinatorial Complexity

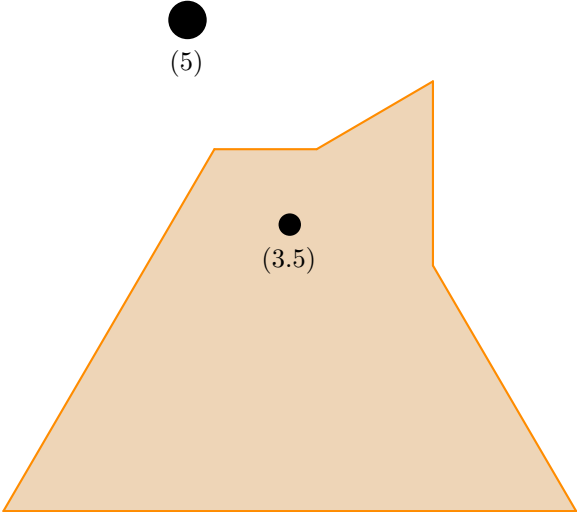


# Voronoi Diagram – Incremental Construction

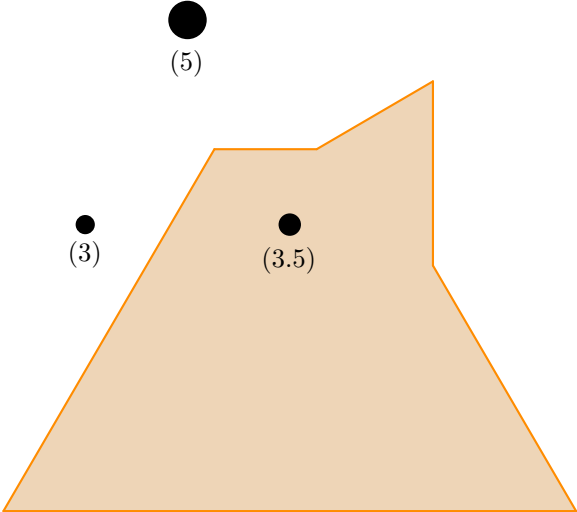


(5)

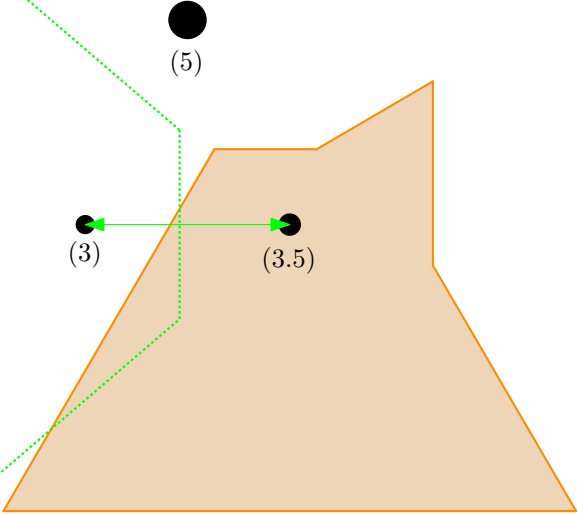
# Voronoi Diagram – Incremental Construction



# Voronoi Diagram – Incremental Construction

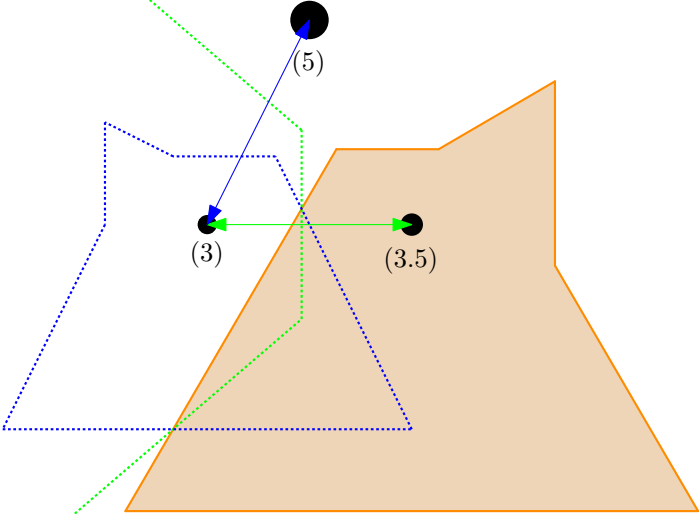


# Voronoi Diagram – Incremental Construction

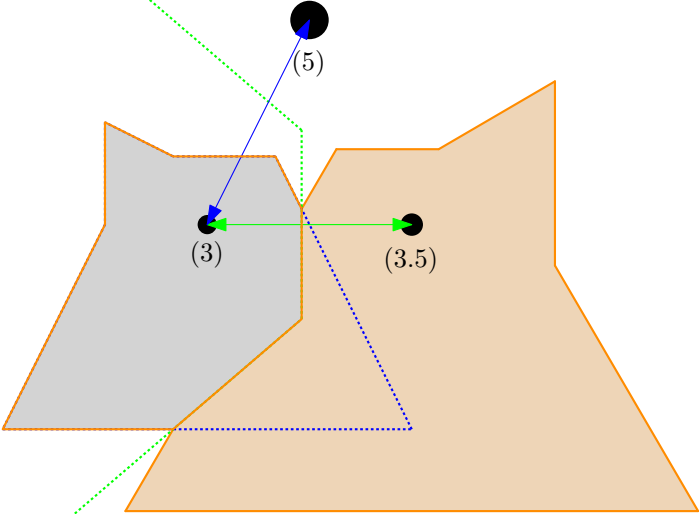




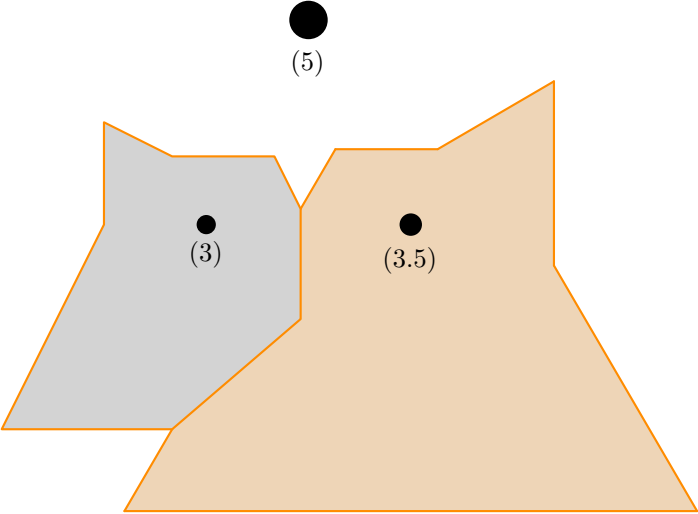
# Voronoi Diagram – Incremental Construction



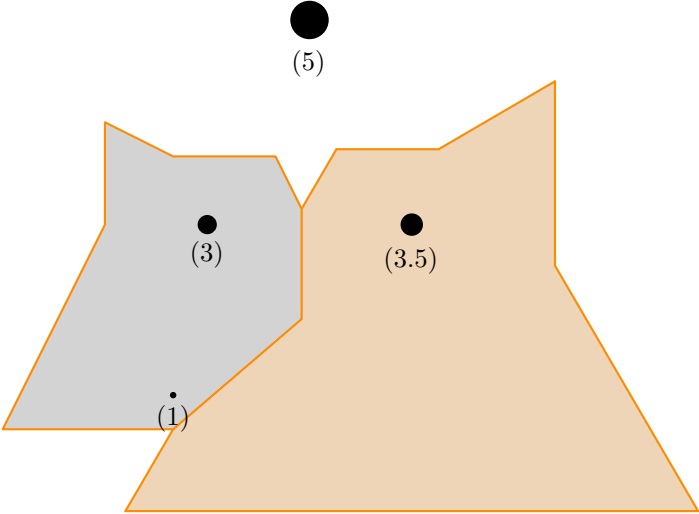
# Voronoi Diagram – Incremental Construction



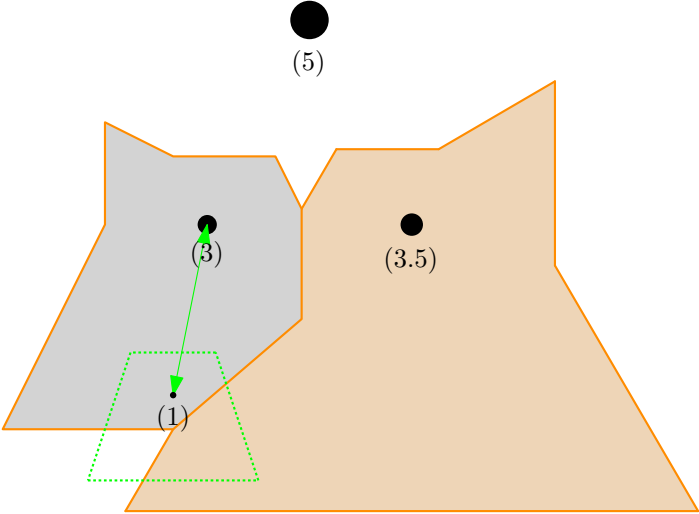
# Voronoi Diagram – Incremental Construction



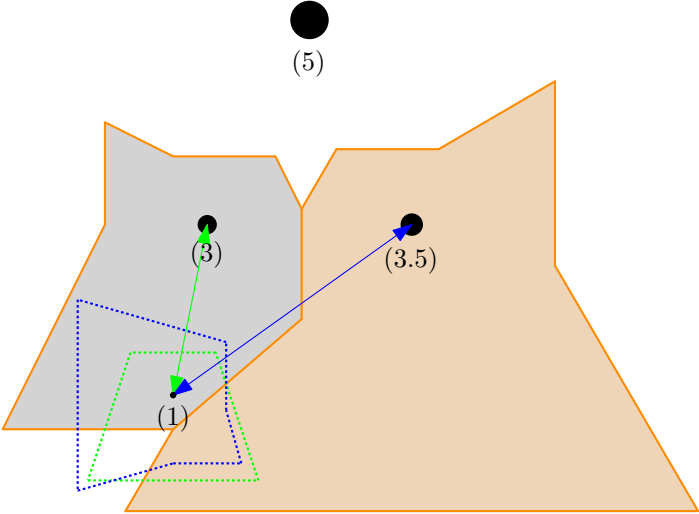
# Voronoi Diagram – Incremental Construction



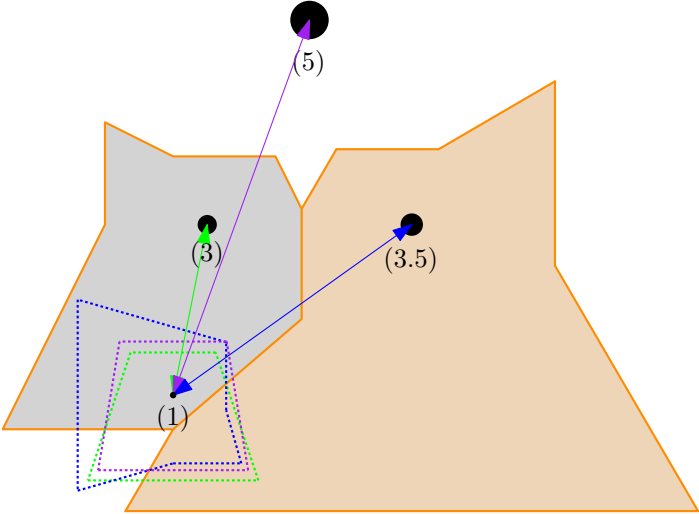
# Voronoi Diagram – Incremental Construction



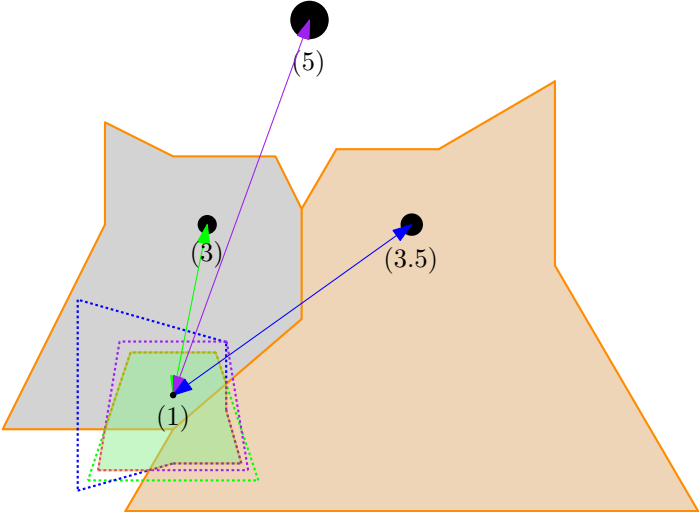
# Voronoi Diagram – Incremental Construction



# Voronoi Diagram – Incremental Construction

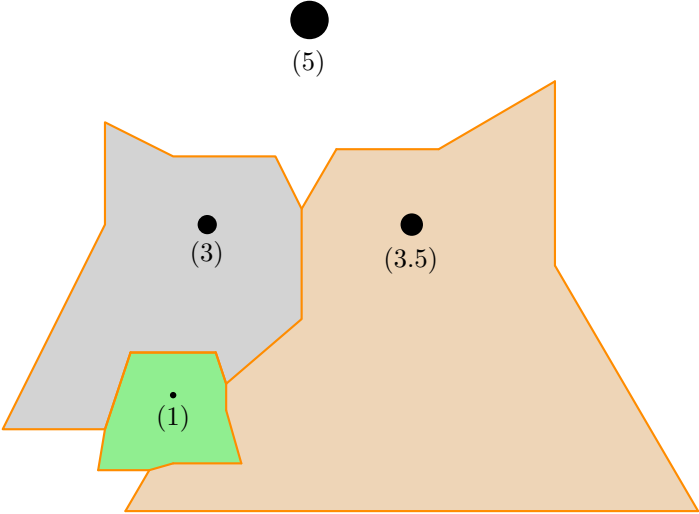


# Voronoi Diagram – Incremental Construction

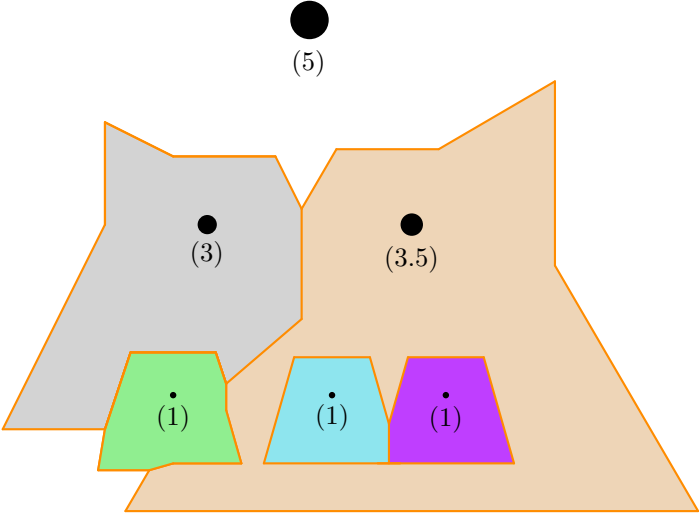




# Voronoi Diagram – Incremental Construction

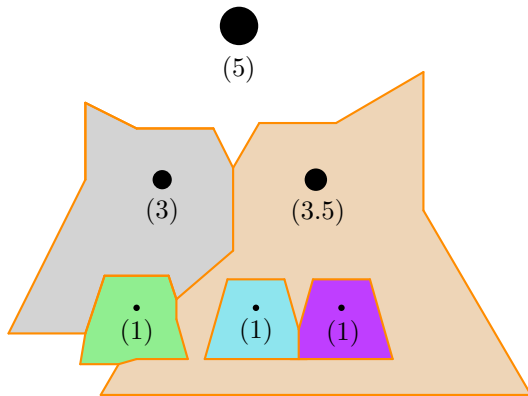


# Voronoi Diagram – Incremental Construction



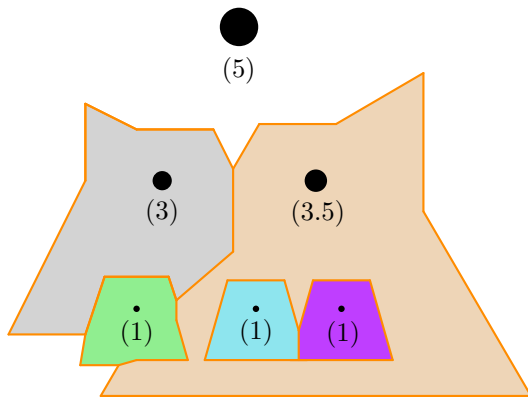
## Voronoi Diagram – Summary

- Compute single region –  $\mathcal{O}(n \log n)$
- Embed region in diagram –  $\mathcal{O}(n \log n)$
- Overall  $\mathcal{O}(n^2 \log n)$  time and  $\mathcal{O}(n^2)$  space.



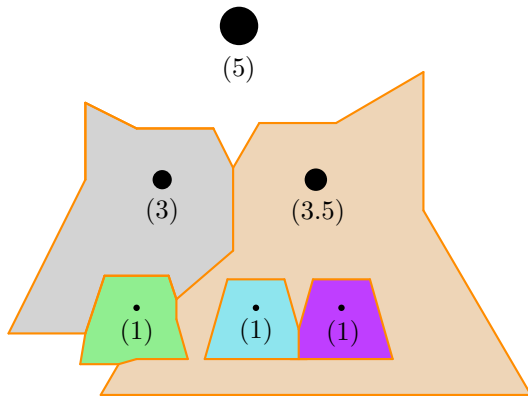
## Voronoi Diagram – Summary

- Compute single region –  $\mathcal{O}(n \log n)$
- Embed region in diagram –  $\mathcal{O}(n \log n)$
- Overall  $\mathcal{O}(n^2 \log n)$  time and  $\mathcal{O}(n^2)$  space.

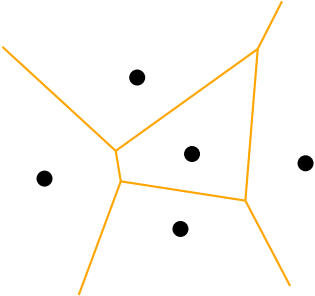


## Voronoi Diagram – Summary

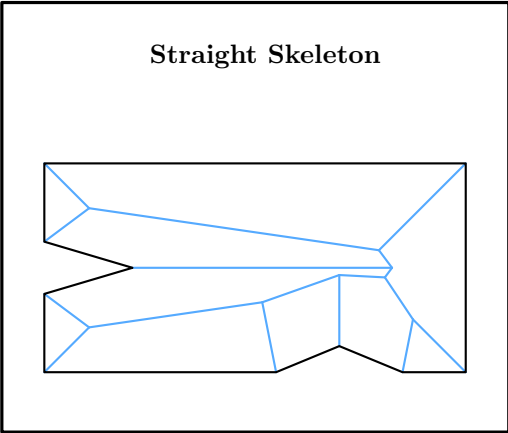
- Compute single region –  $\mathcal{O}(n \log n)$
- Embed region in diagram –  $\mathcal{O}(n \log n)$
- Overall  $\mathcal{O}(n^2 \log n)$  time and  $\mathcal{O}(n^2)$  space.



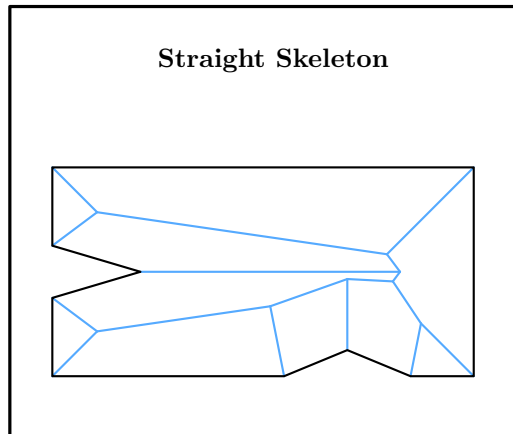
## Voronoi Diagram



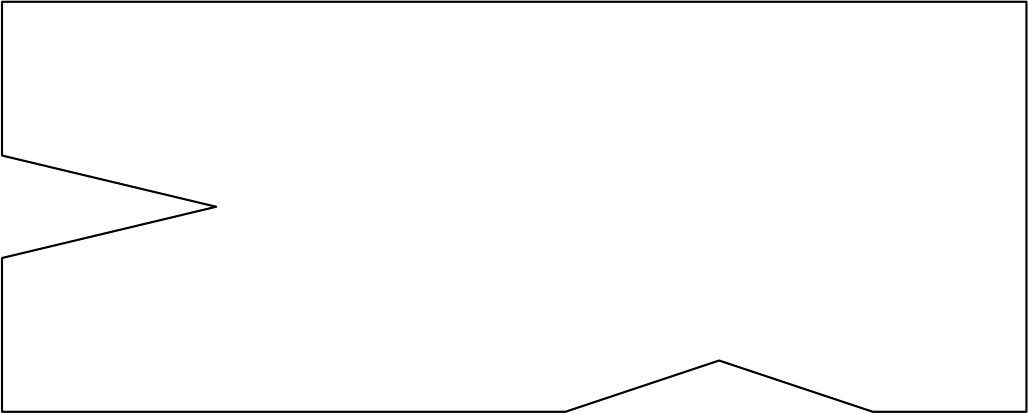
## Straight Skeleton



- Definition
- Applications
- Our Result

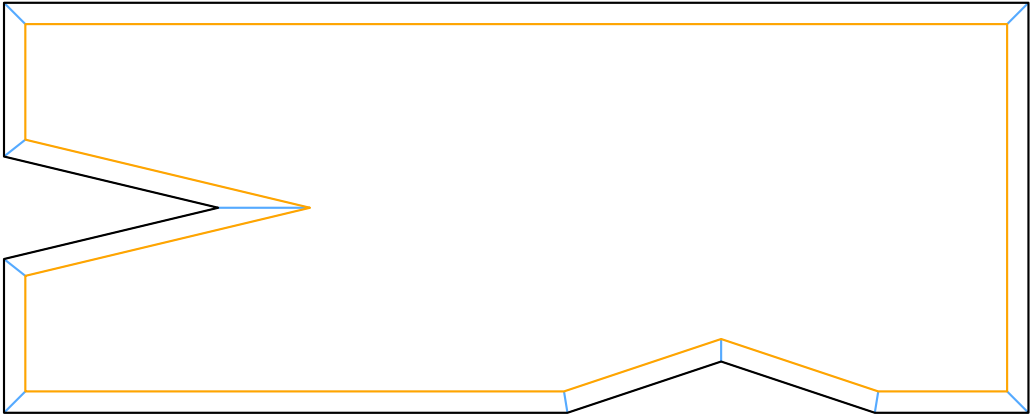


# Straight Skeleton – Definition



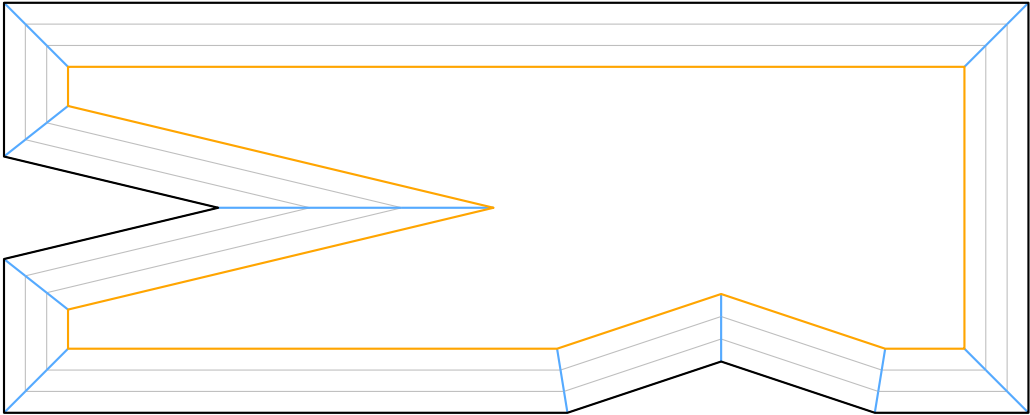


# Straight Skeleton – Definition

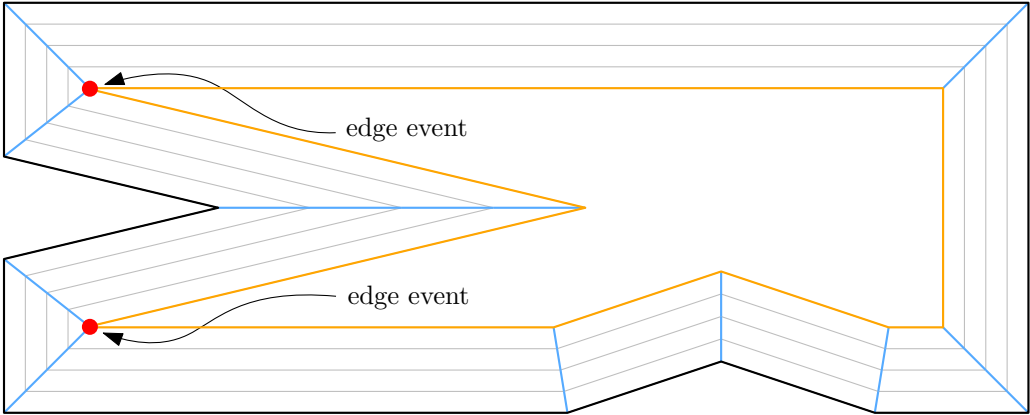




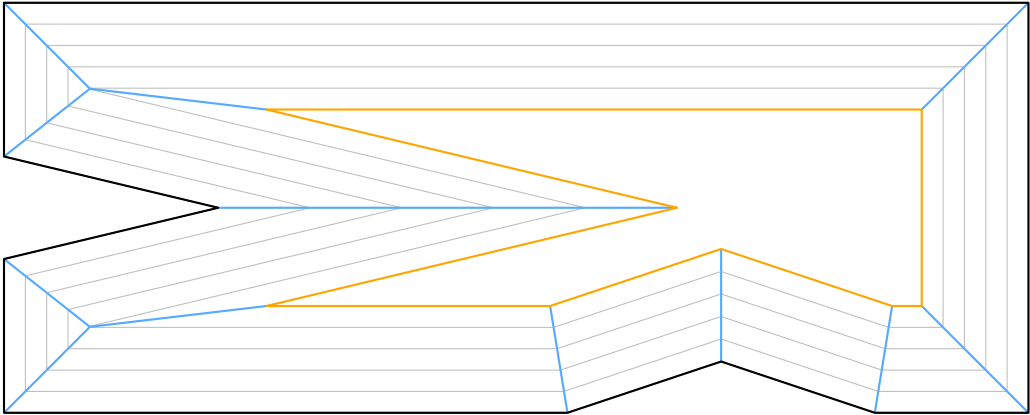
# Straight Skeleton – Definition



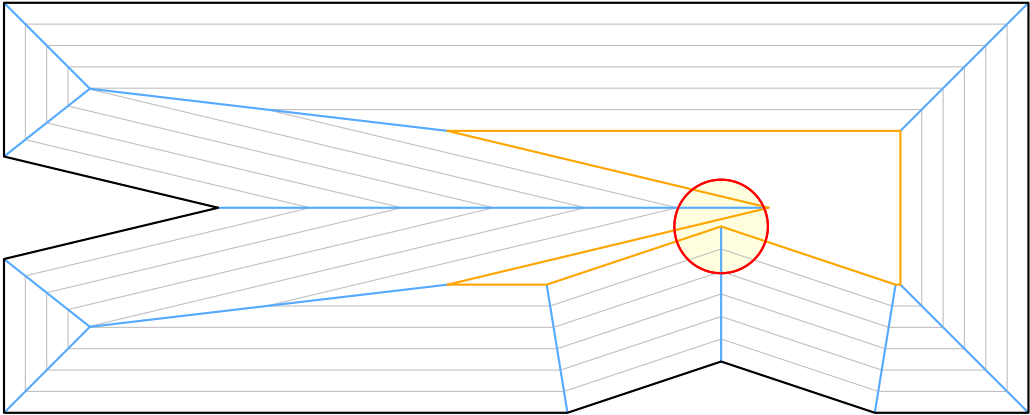
# Straight Skeleton – Definition



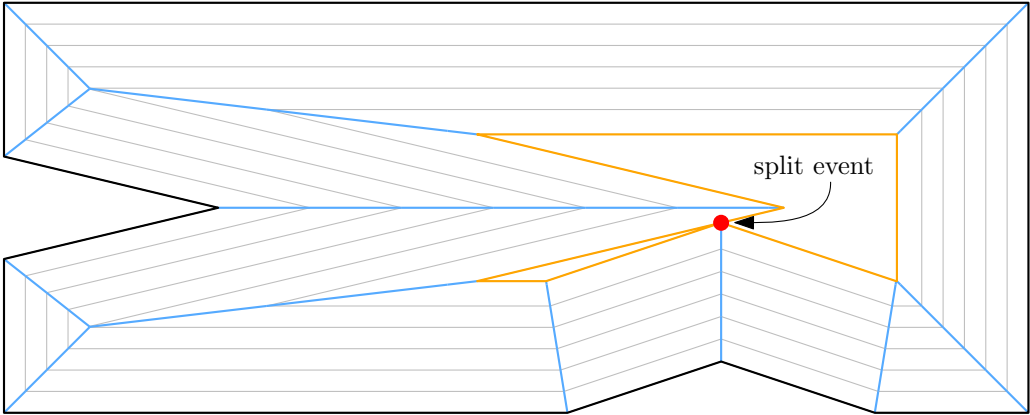
# Straight Skeleton – Definition



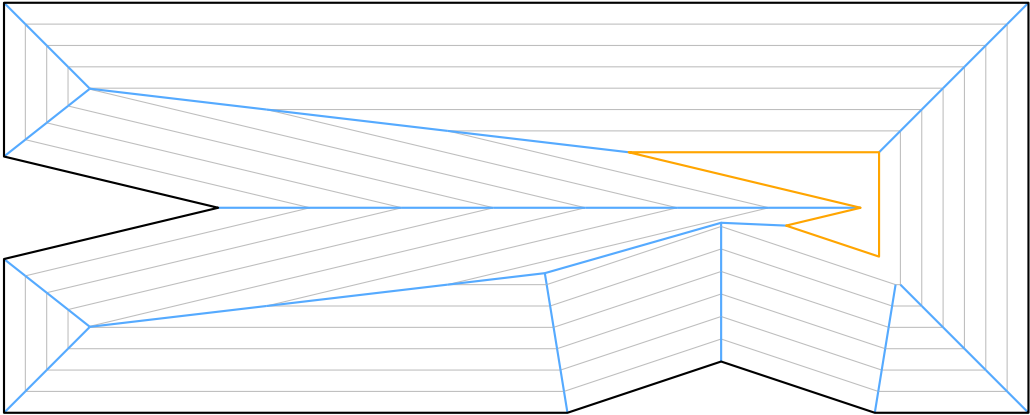
# Straight Skeleton – Definition



# Straight Skeleton – Definition

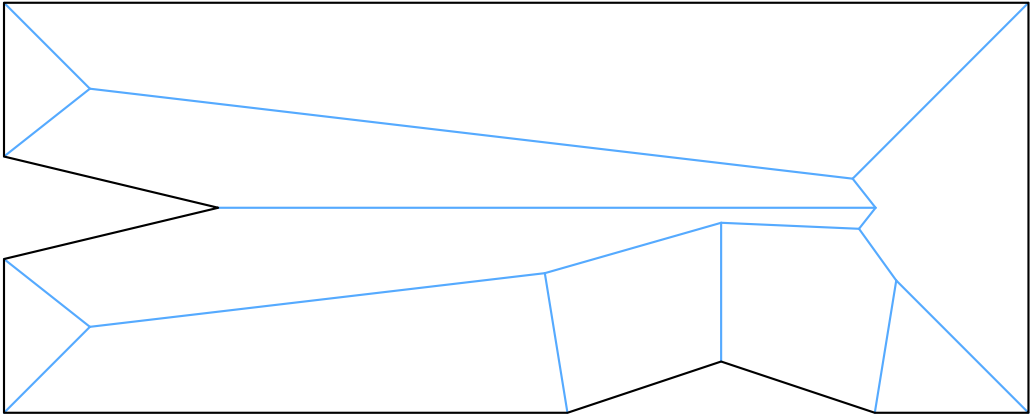


# Straight Skeleton – Definition

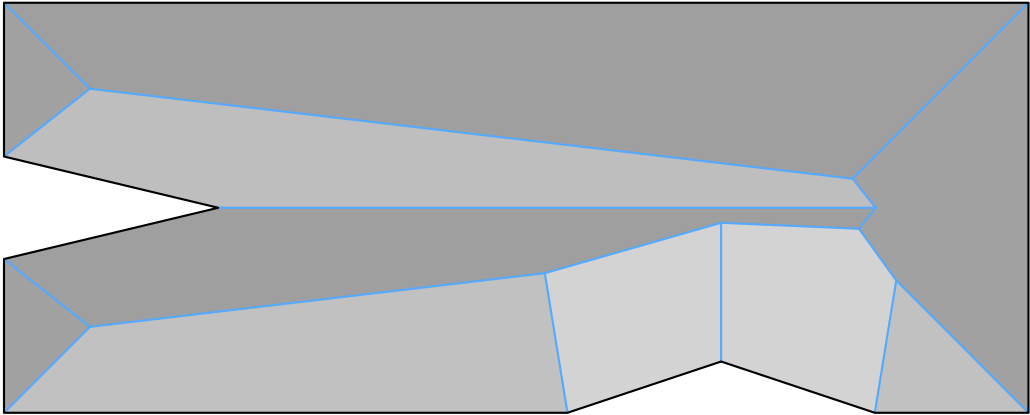




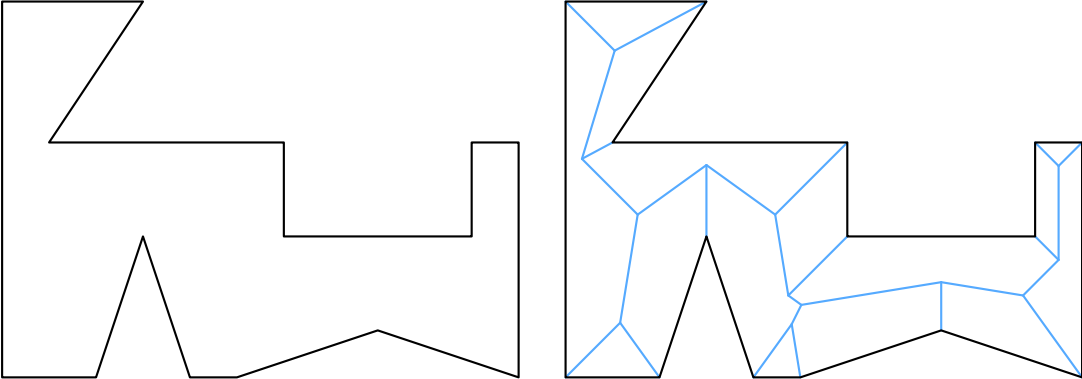
# Straight Skeleton – Definition



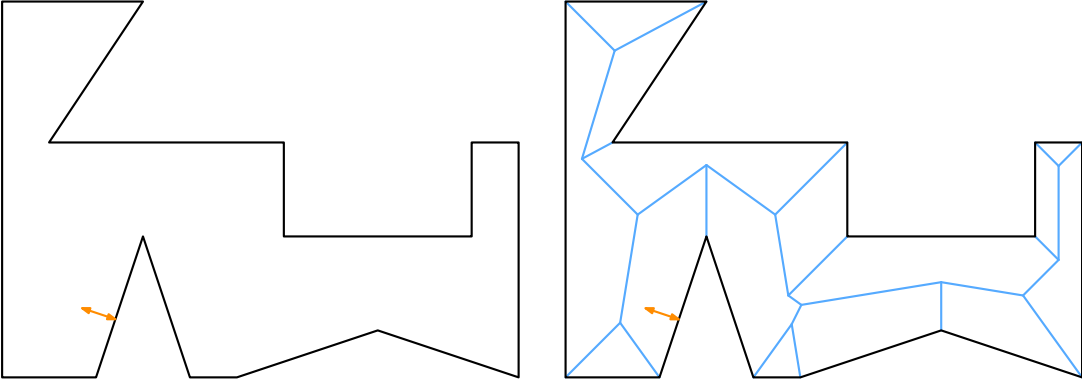
# Straight Skeleton – Definition



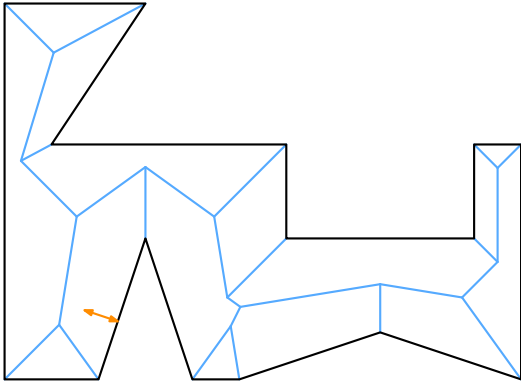
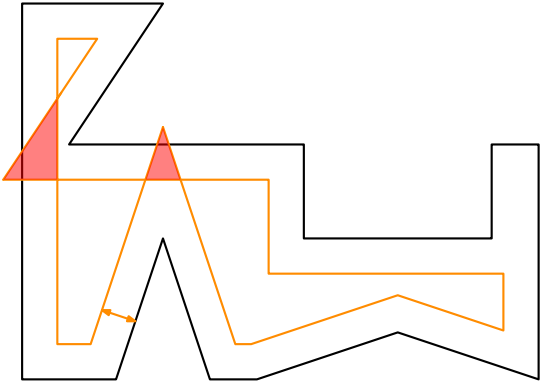
# Straight Skeleton – Application: Polygon Offsetting



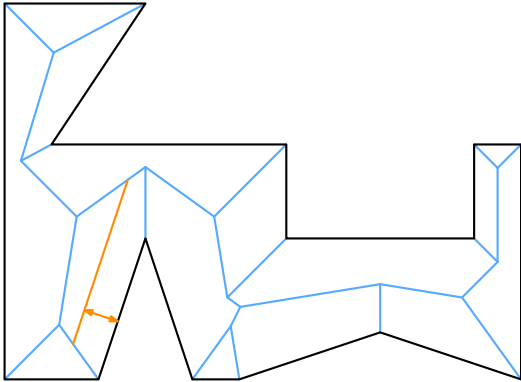
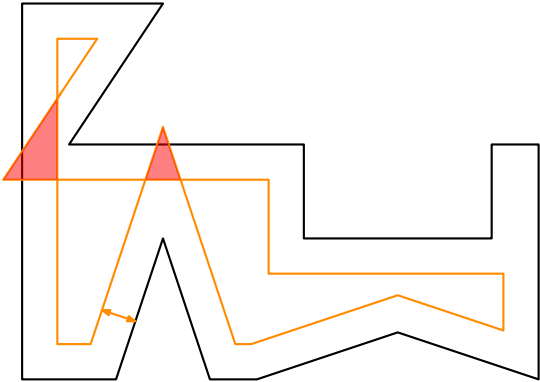
# Straight Skeleton – Application: Polygon Offsetting



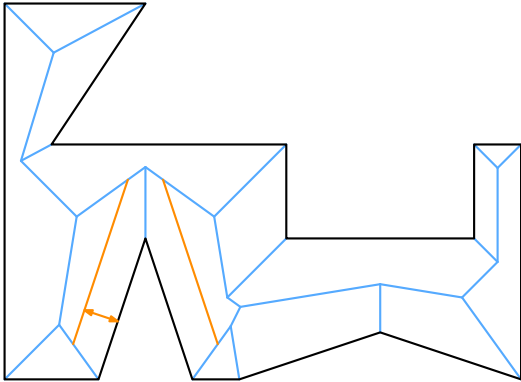
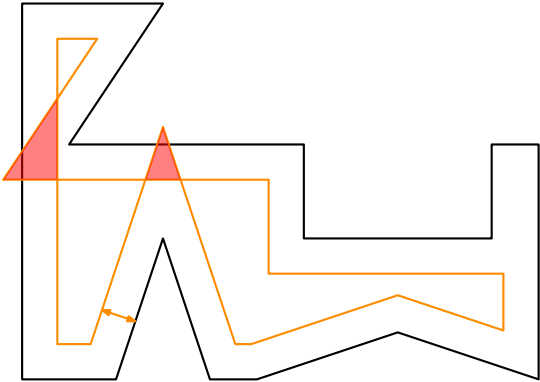
# Straight Skeleton – Application: Polygon Offsetting



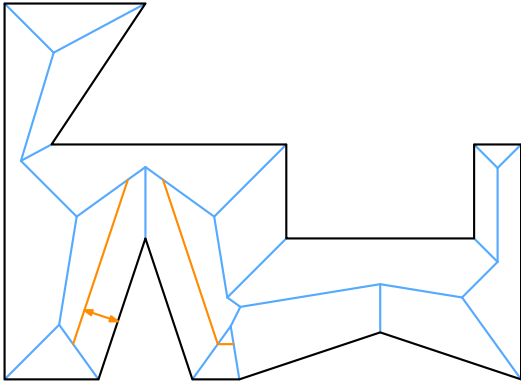
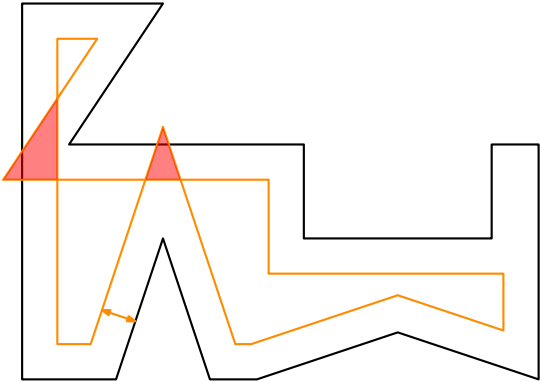
# Straight Skeleton – Application: Polygon Offsetting



# Straight Skeleton – Application: Polygon Offsetting

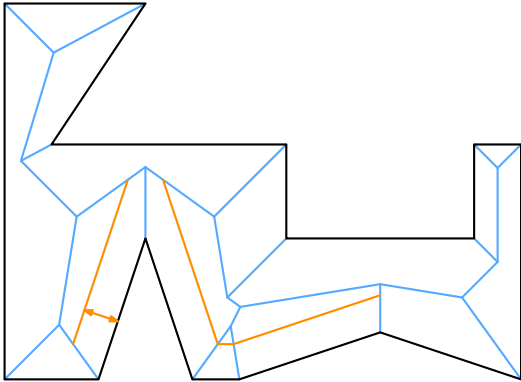
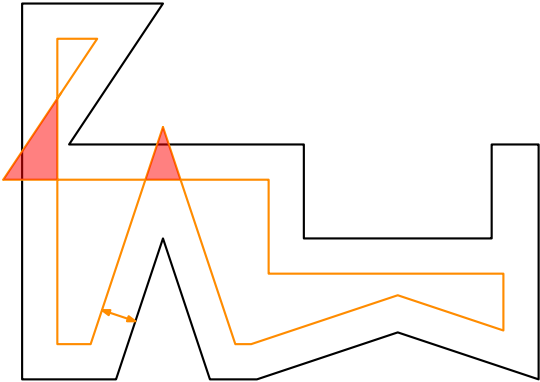


# Straight Skeleton – Application: Polygon Offsetting

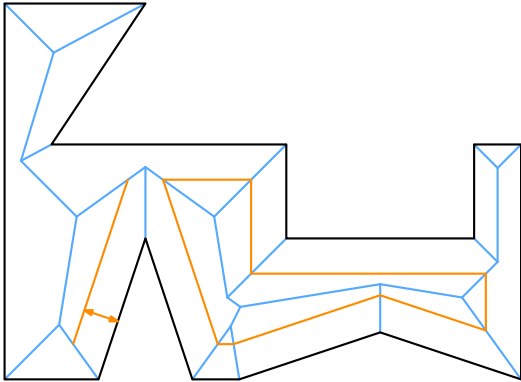
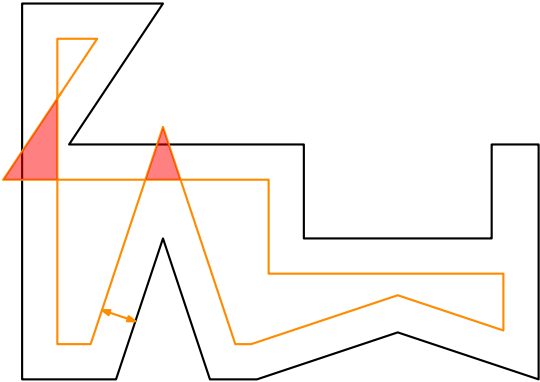




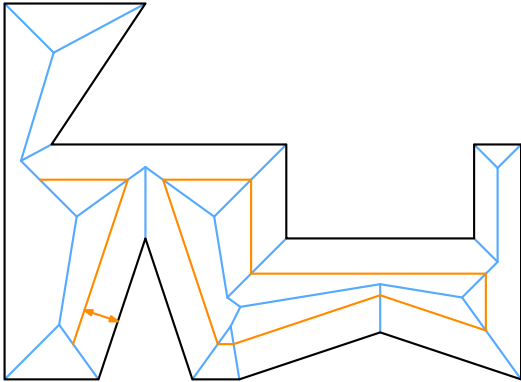
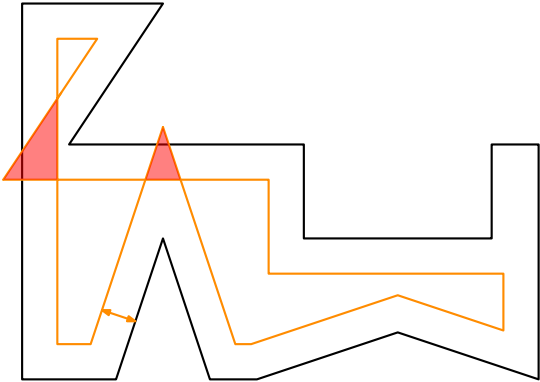
# Straight Skeleton – Application: Polygon Offsetting



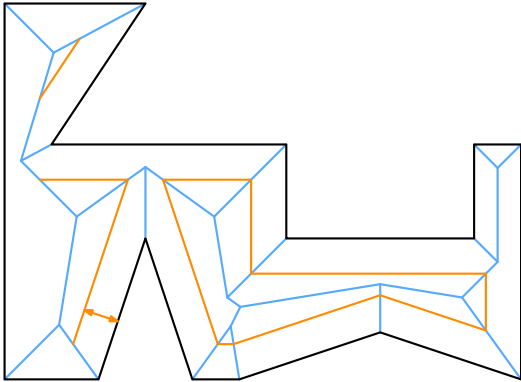
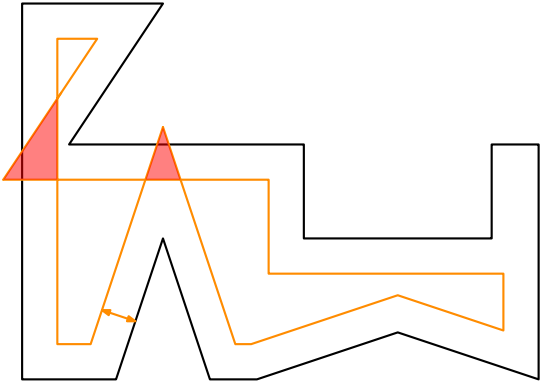
# Straight Skeleton – Application: Polygon Offsetting



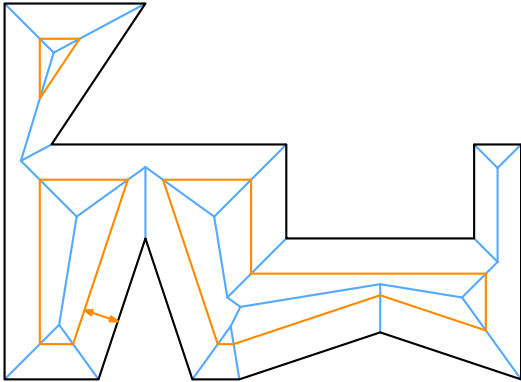
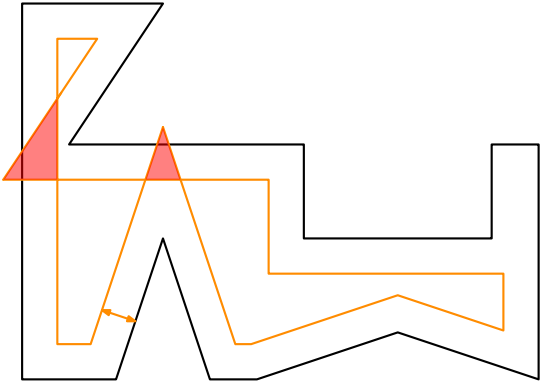
# Straight Skeleton – Application: Polygon Offsetting



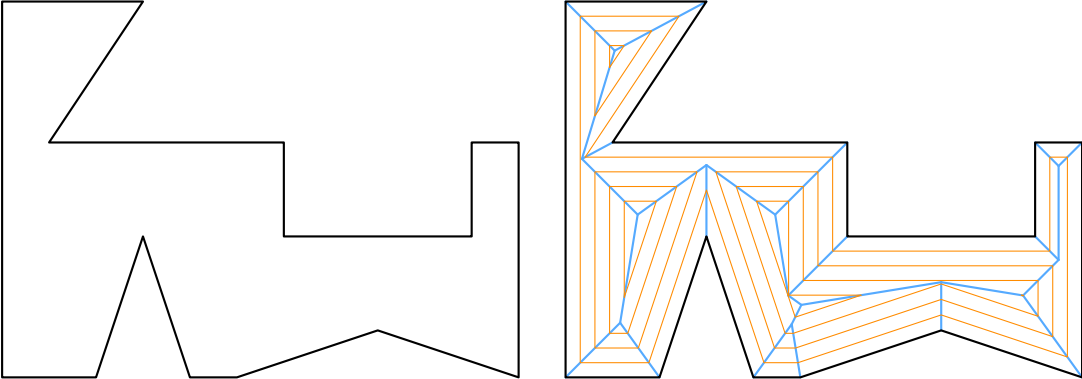
# Straight Skeleton – Application: Polygon Offsetting



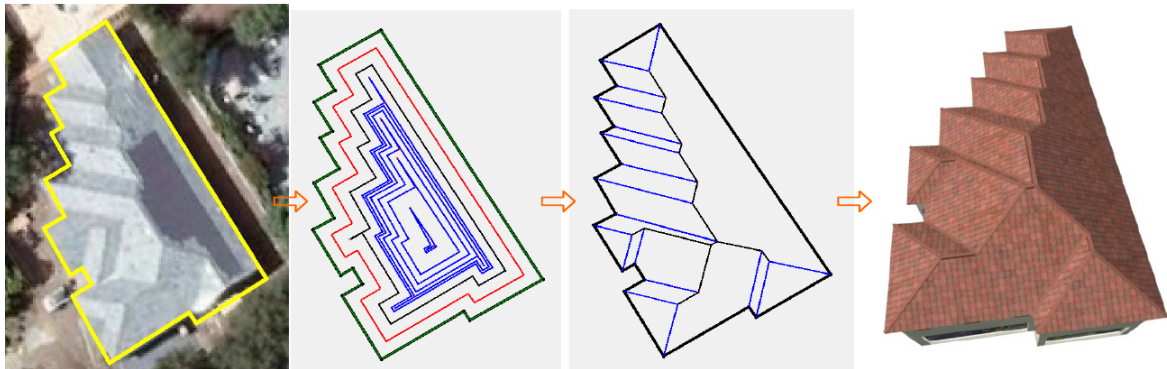
# Straight Skeleton – Application: Polygon Offsetting



# Straight Skeleton – Application: Polygon Offsetting

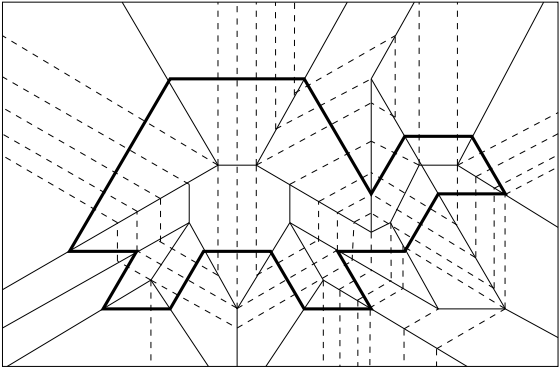
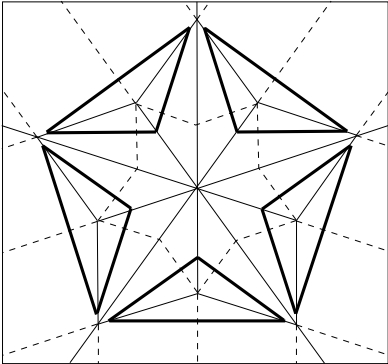


# Straight Skeleton – Application: Automated Roof Construction



[5] Sugihara and Khmelevsky - Roof report from automatically generated 3D building models by straight skeleton computation

# Straight Skeleton – Application: Mathematical Origami

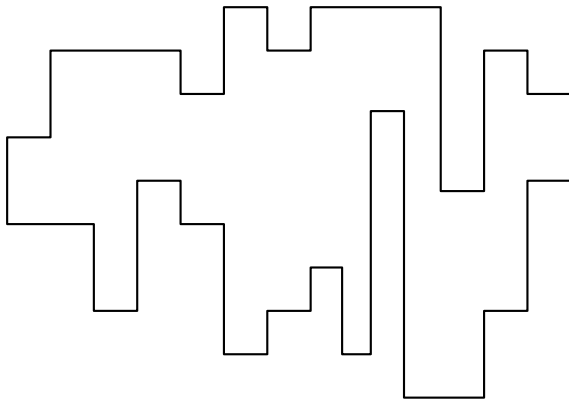


[2] Demaine et al. - Folding and One Straight Cut Suffice



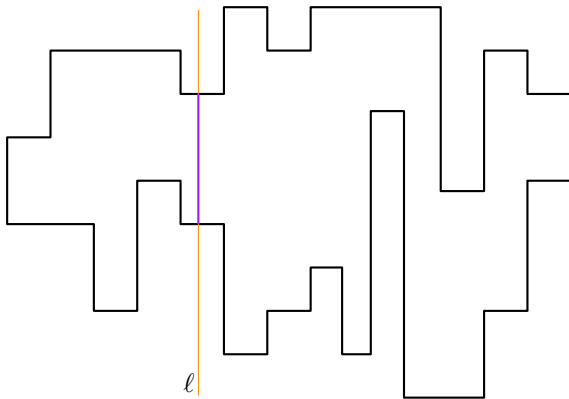
## Straight Skeleton – Our Result

- Straight skeleton of an orthogonal monotone polygon.
- Linear time algorithm.
- Best known approaches:
  - Straight skeleton –  $\mathcal{O}(n^{17/11+\epsilon})$  [3]
  - Voronoi diagram  $L_\infty$  –  $\mathcal{O}(n \log n)$  [4]



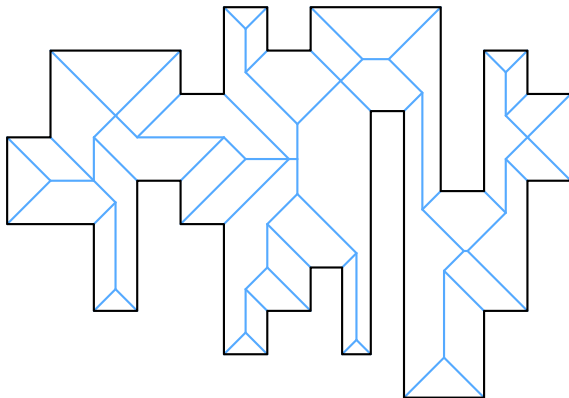
## Straight Skeleton – Our Result

- Straight skeleton of an orthogonal  $x$ -monotone polygon.
- Linear time algorithm.
- Best known approaches:
  - Straight skeleton –  $\mathcal{O}(n^{17/11+\epsilon})$  [3]
  - Voronoi diagram  $L_\infty$  –  $\mathcal{O}(n \log n)$  [4]



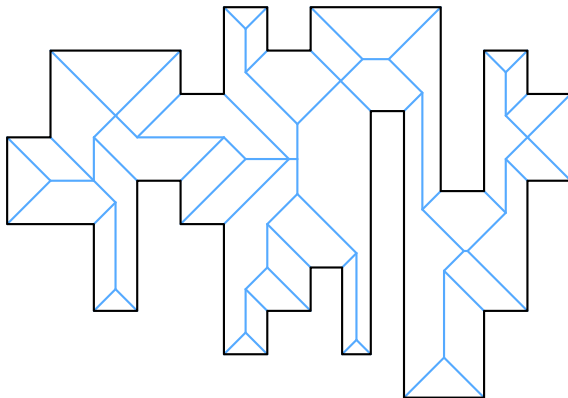
## Straight Skeleton – Our Result

- Straight skeleton of an orthogonal  $x$ -monotone polygon.
- Linear time algorithm.
- Best known approaches:
  - Straight skeleton –  $\mathcal{O}(n^{17/11+\epsilon})$  [3]
  - Voronoi diagram  $L_\infty$  –  $\mathcal{O}(n \log n)$  [4]



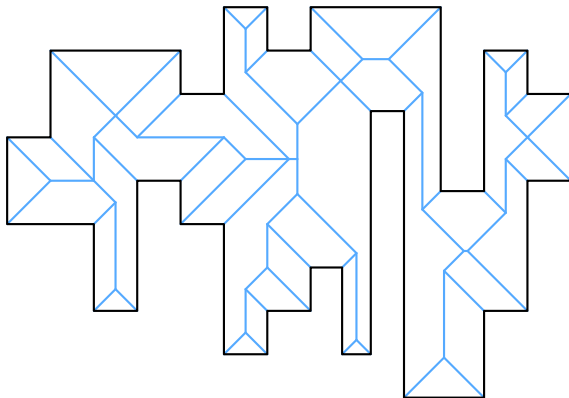
## Straight Skeleton – Our Result

- Straight skeleton of an orthogonal  $x$ -monotone polygon.
- Linear time algorithm.
- Best known approaches:
  - Straight skeleton –  $\mathcal{O}(n^{17/11+\epsilon})$  [3]
  - Voronoi diagram  $L_\infty$  –  $\mathcal{O}(n \log n)$  [4]



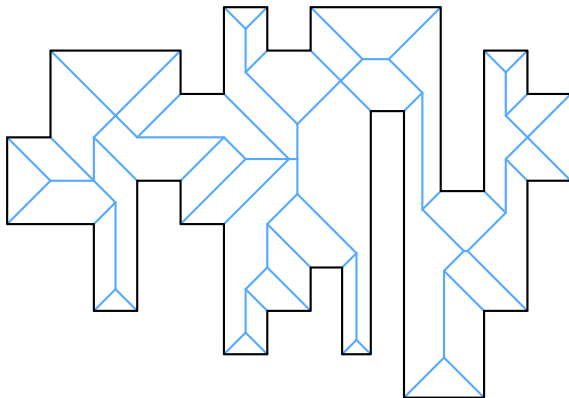
## Straight Skeleton – Our Result

- Straight skeleton of an orthogonal  $x$ -monotone polygon.
- Linear time algorithm.
- Best known approaches:
  - Straight skeleton –  $\mathcal{O}(n^{17/11+\epsilon})$  [3]
  - Voronoi diagram  $L_\infty$  –  $\mathcal{O}(n \log n)$  [4]



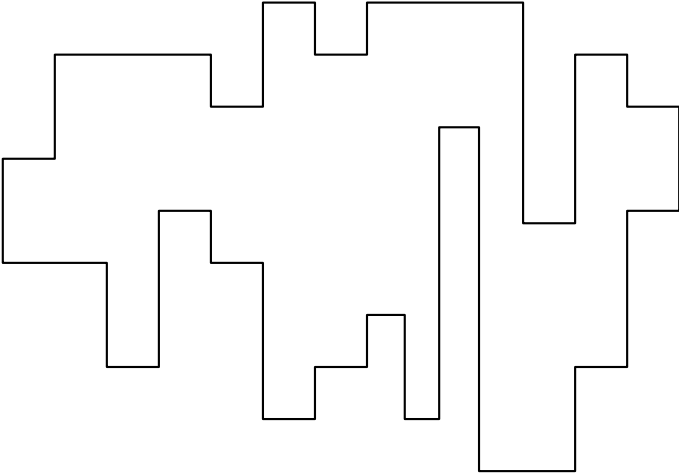
[3] Eppstein and Erickson - Raising Roofs, Crashing Cycles, and Playing Pool

- Straight skeleton of an orthogonal  $x$ -monotone polygon.
- Linear time algorithm.
- Best known approaches:
  - Straight skeleton –  $\mathcal{O}(n^{17/11+\epsilon})$  [3]
  - Voronoi diagram  $L_\infty$  –  $\mathcal{O}(n \log n)$  [4]

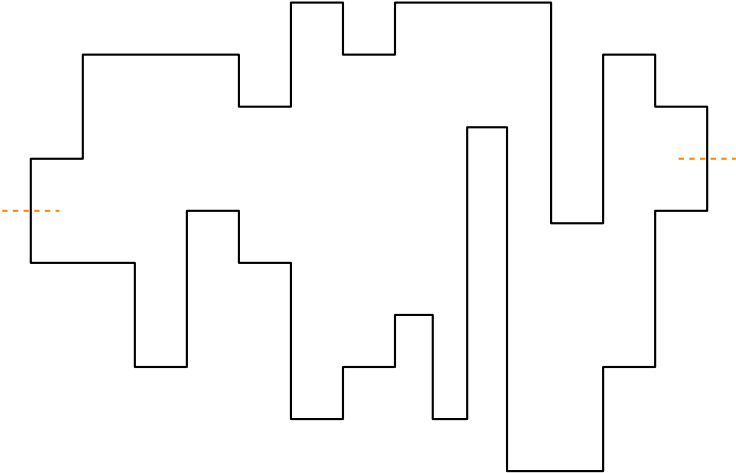


[3] Eppstein and Erickson - Raising Roofs, Crashing Cycles, and Playing Pool – [4] Papadopoulou et al. - The  $L_\infty$  Voronoi Diagram of Segments

# Straight Skeleton – Algorithm

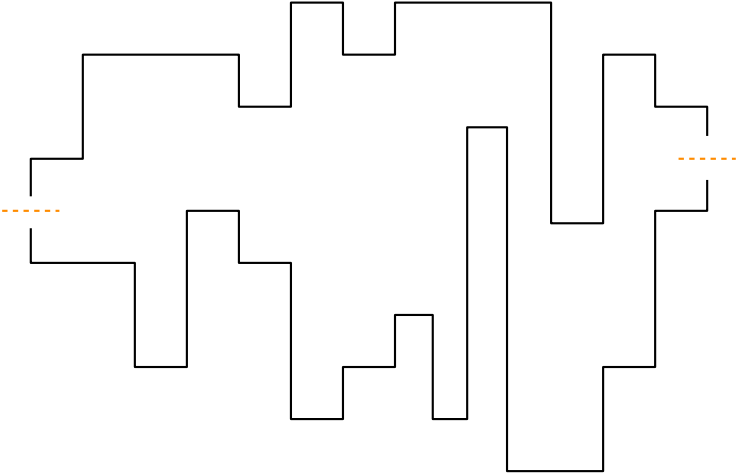


# Straight Skeleton – Algorithm

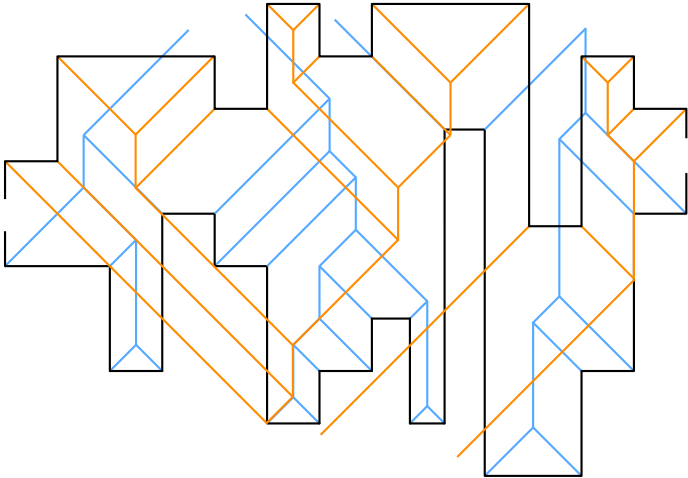




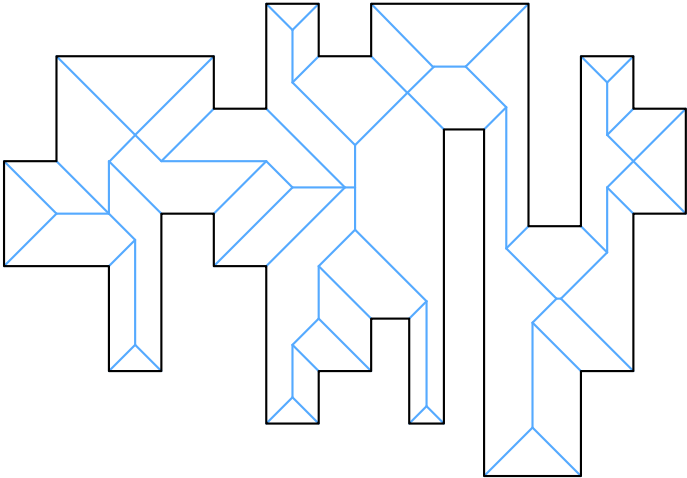
# Straight Skeleton – Algorithm



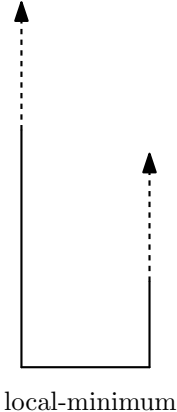
# Straight Skeleton – Algorithm



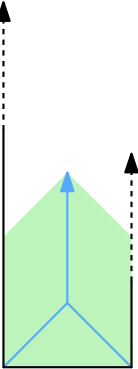
# Straight Skeleton – Algorithm



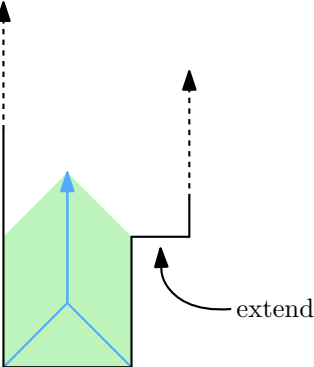
# Straight Skeleton – Algorithm



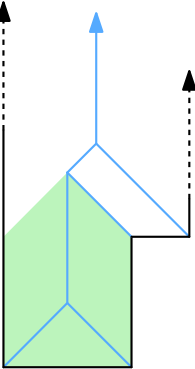
# Straight Skeleton – Algorithm



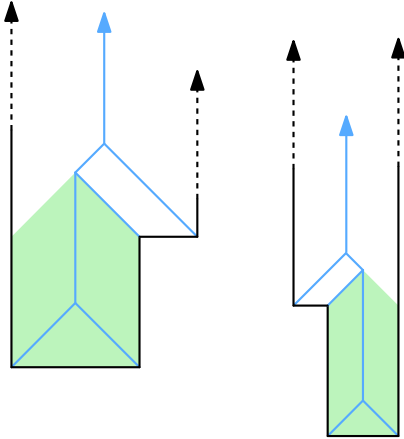
# Straight Skeleton – Algorithm



# Straight Skeleton – Algorithm

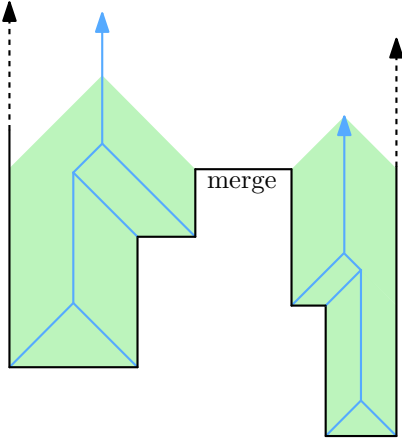


# Straight Skeleton – Algorithm

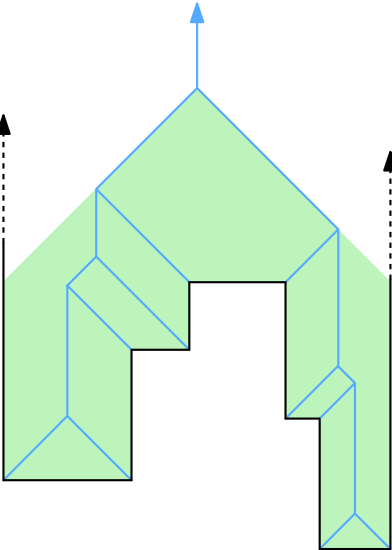




# Straight Skeleton – Algorithm

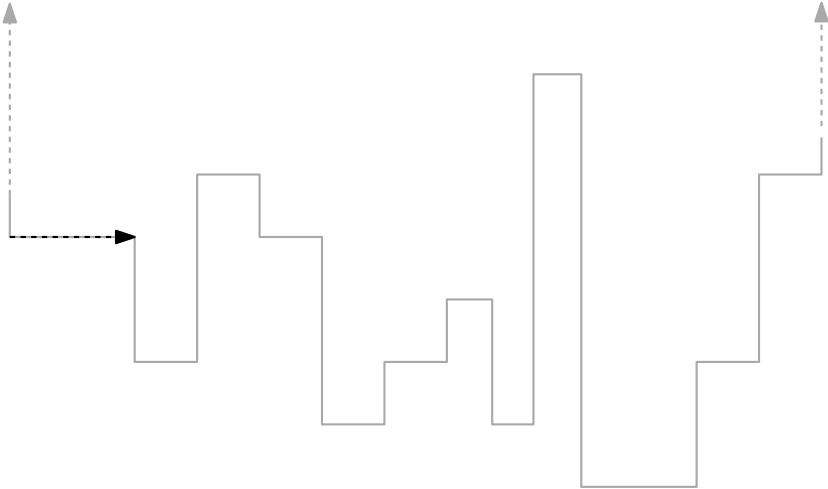


# Straight Skeleton – Algorithm

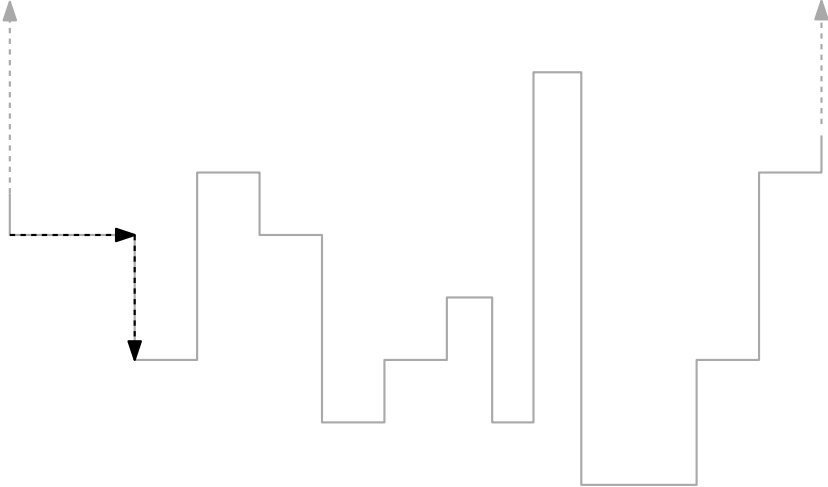




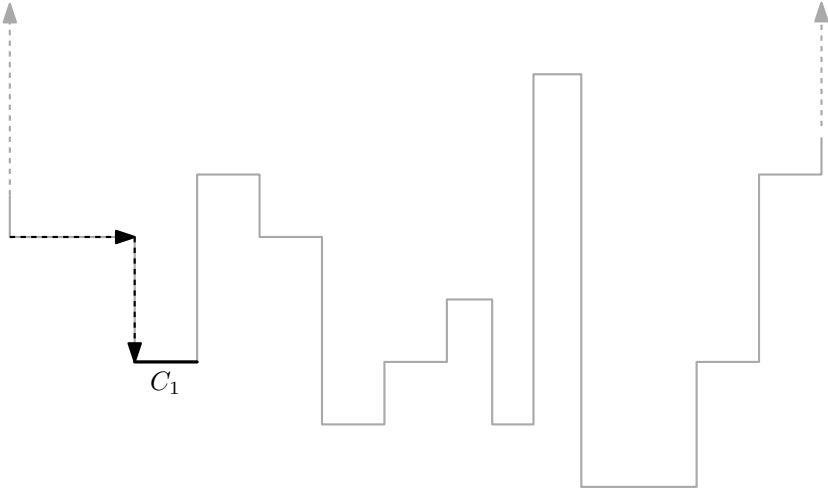
# Straight Skeleton – Algorithm



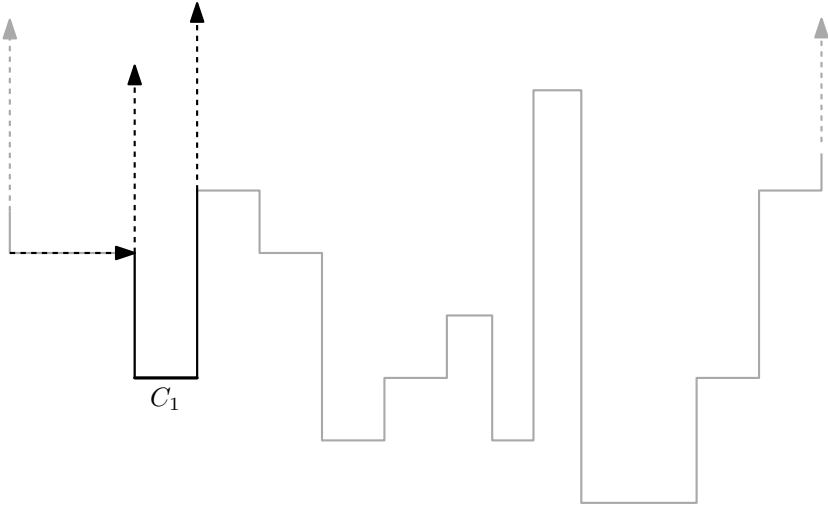
# Straight Skeleton – Algorithm



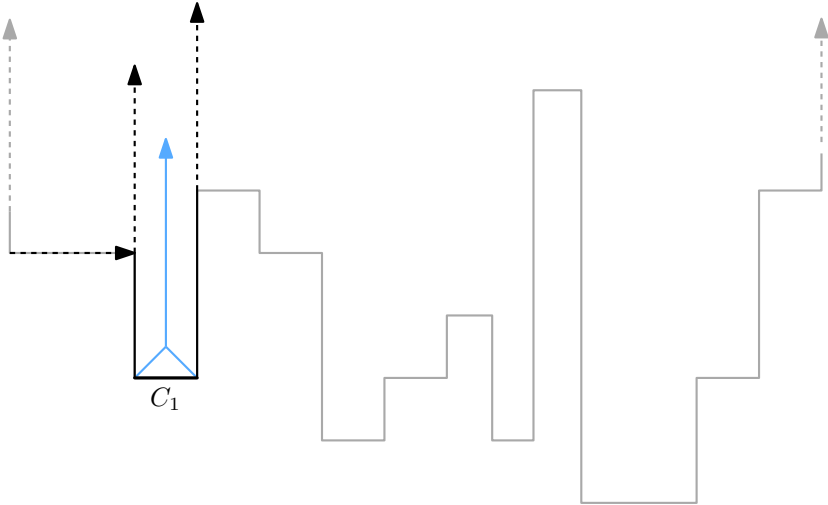
# Straight Skeleton – Algorithm



# Straight Skeleton – Algorithm

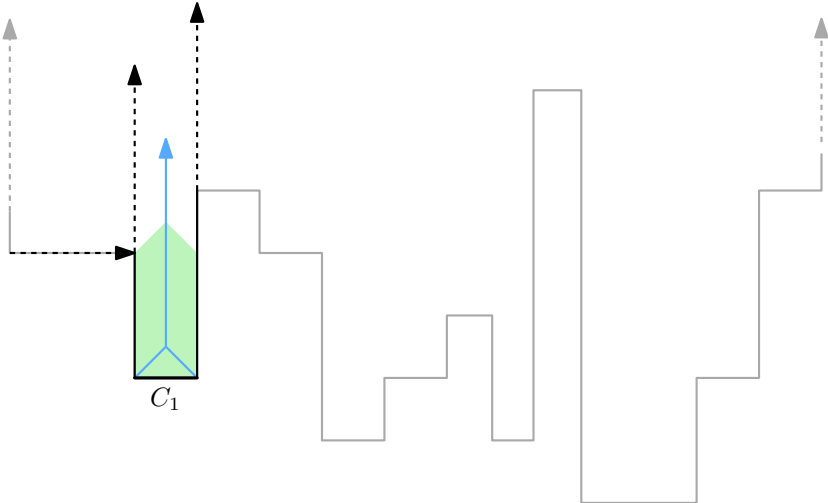


# Straight Skeleton – Algorithm

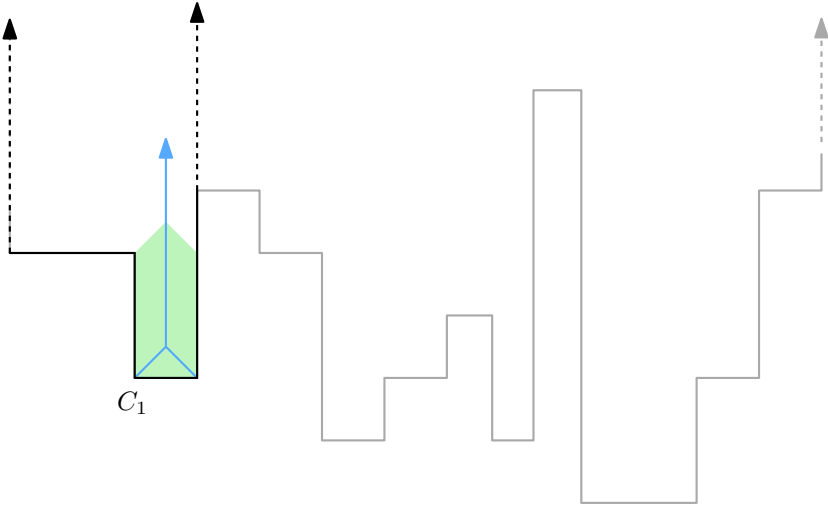




# Straight Skeleton – Algorithm

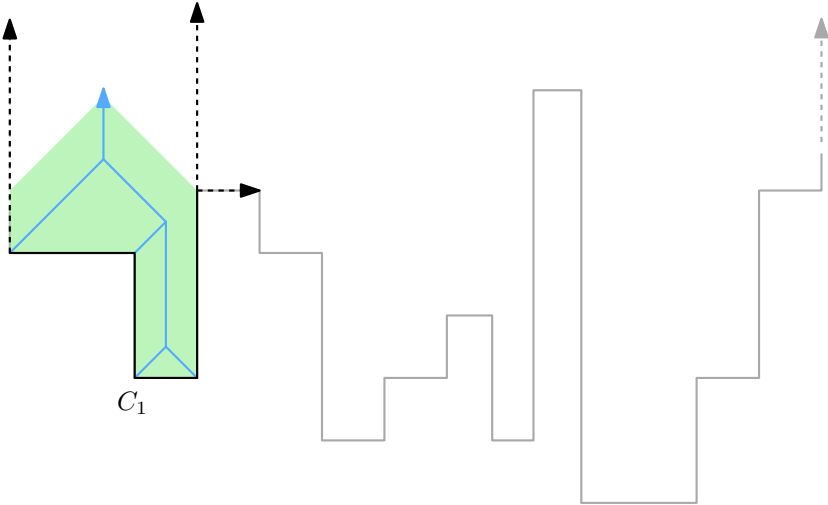


# Straight Skeleton – Algorithm

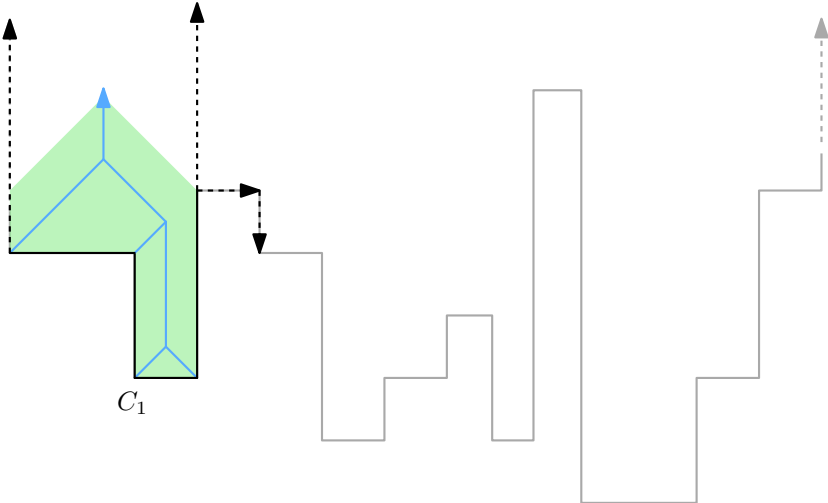




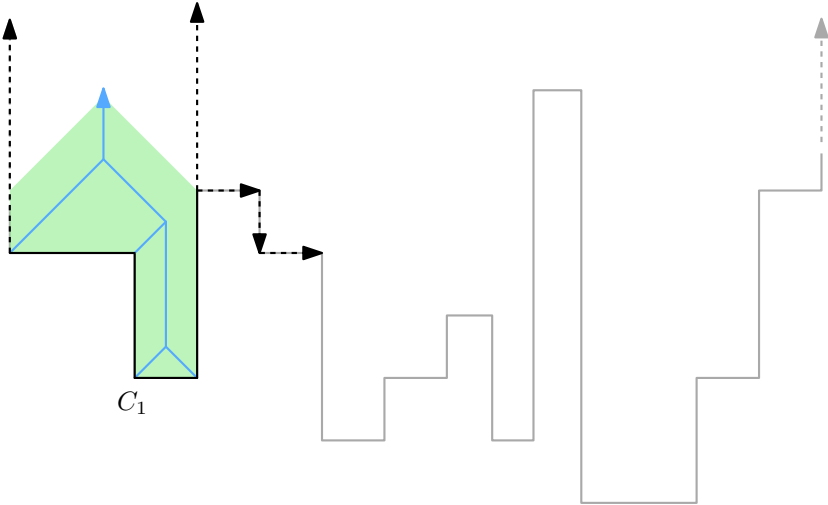
# Straight Skeleton – Algorithm



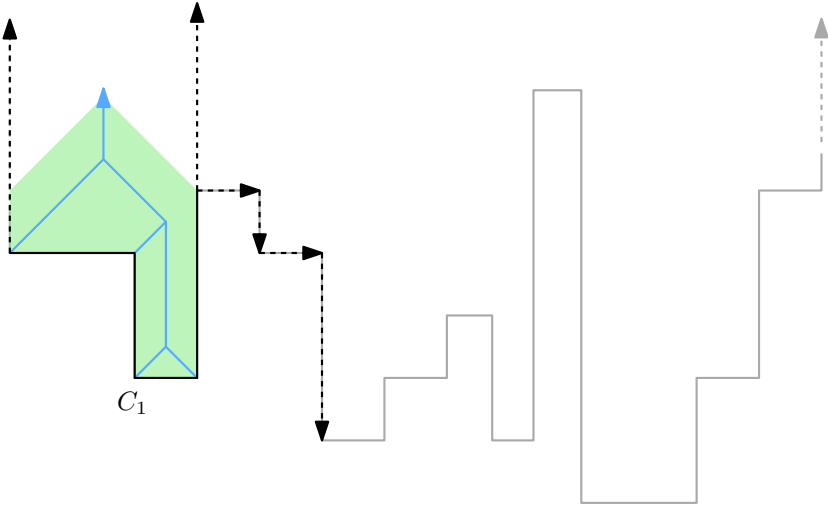
# Straight Skeleton – Algorithm



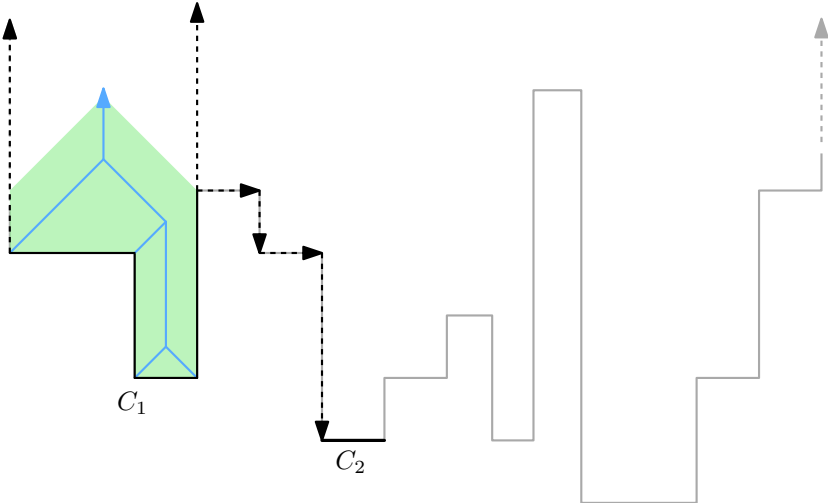
# Straight Skeleton – Algorithm



# Straight Skeleton – Algorithm

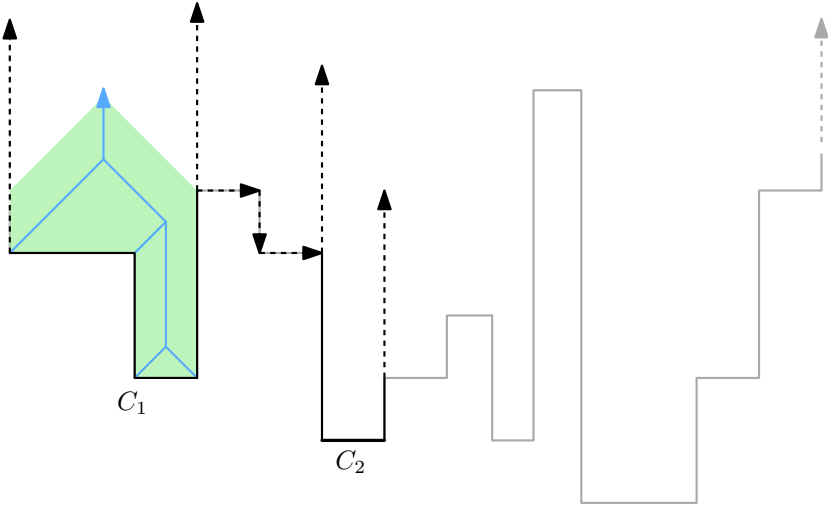


# Straight Skeleton – Algorithm

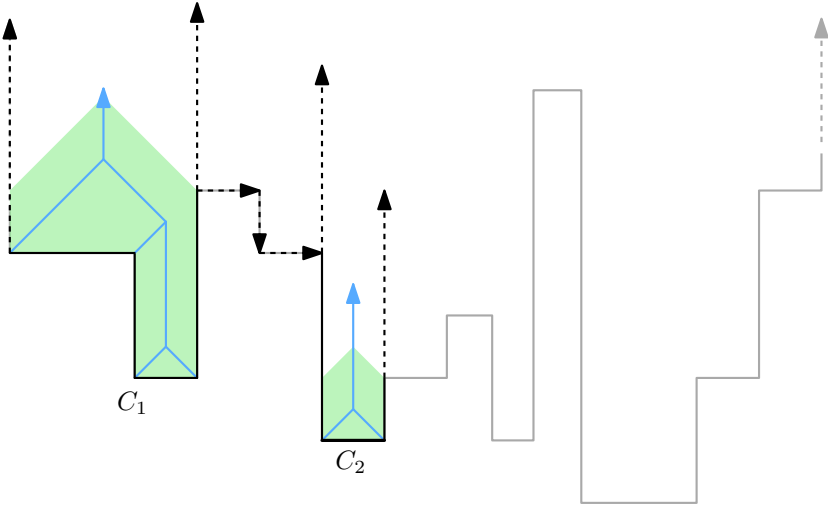




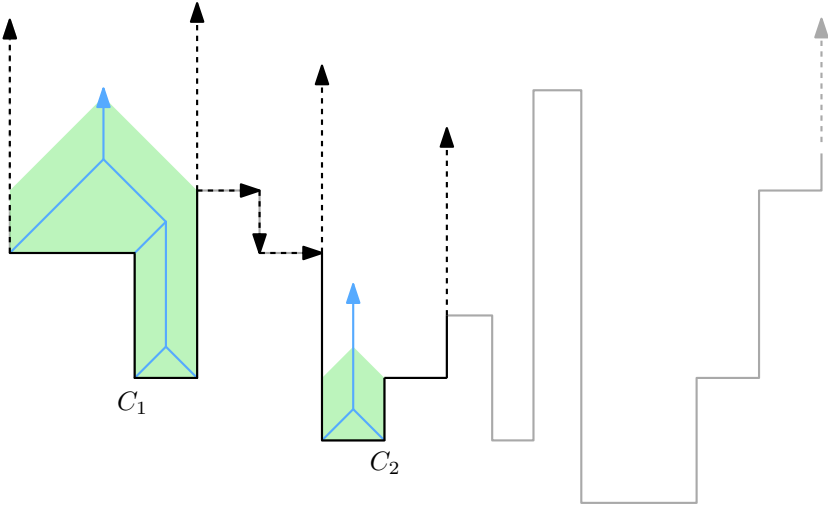
# Straight Skeleton – Algorithm



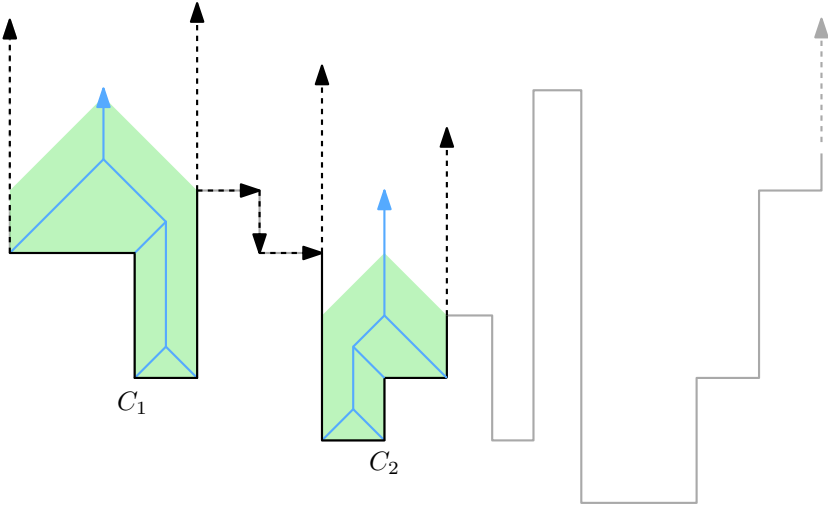
# Straight Skeleton – Algorithm



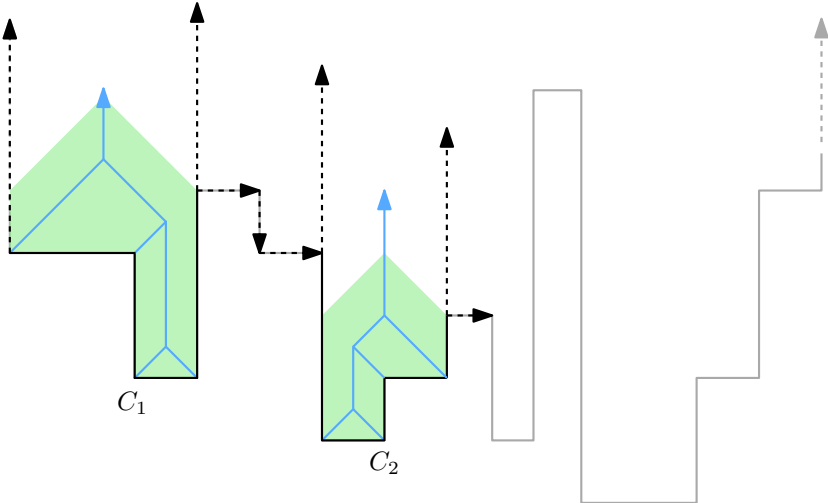
# Straight Skeleton – Algorithm



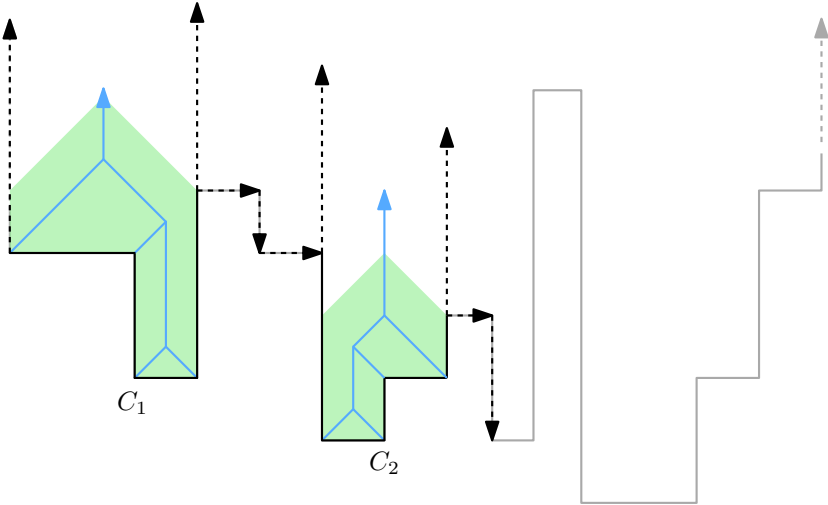
# Straight Skeleton – Algorithm



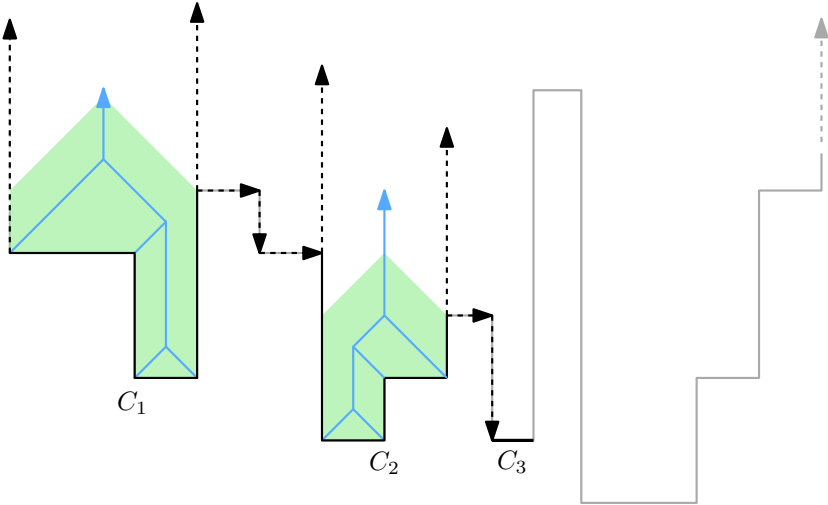
# Straight Skeleton – Algorithm



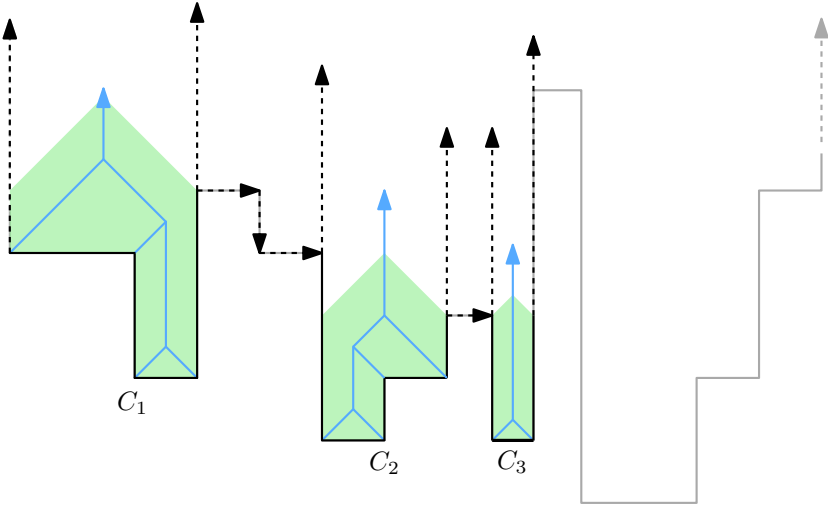
# Straight Skeleton – Algorithm



# Straight Skeleton – Algorithm

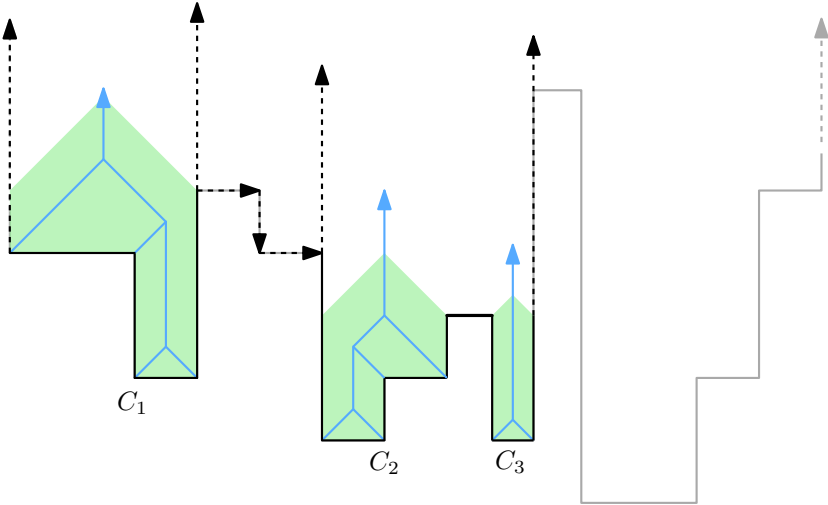


# Straight Skeleton – Algorithm

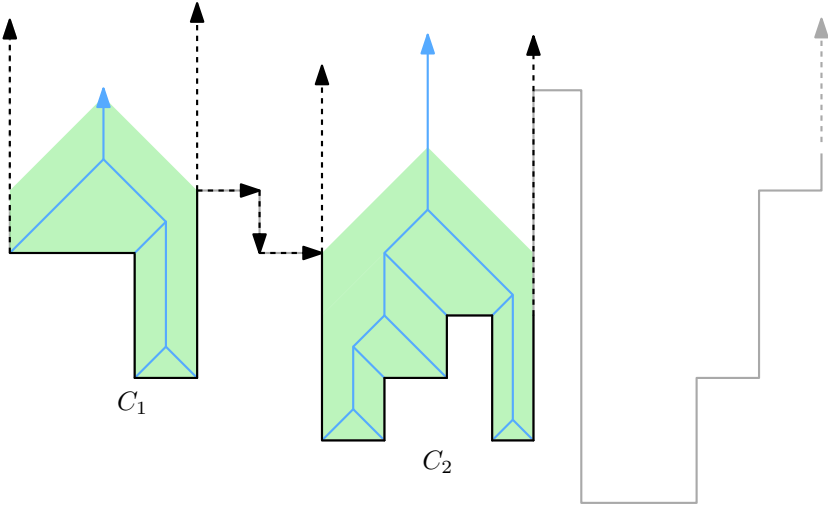




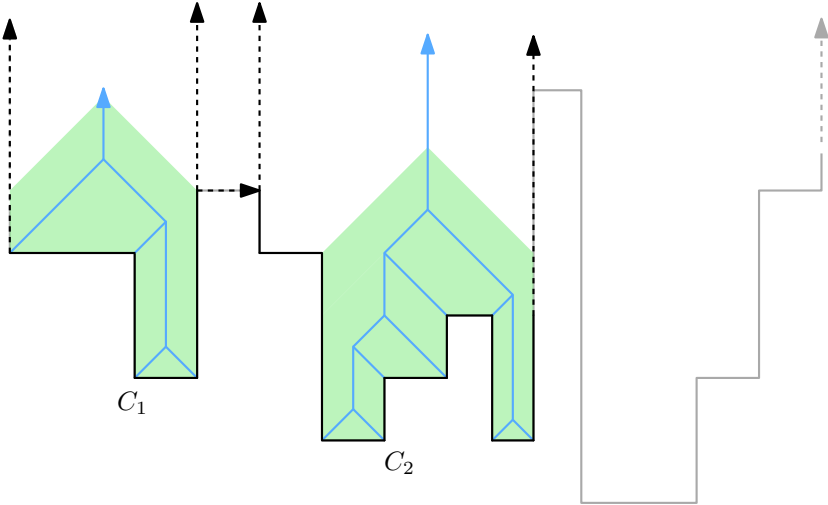
# Straight Skeleton – Algorithm



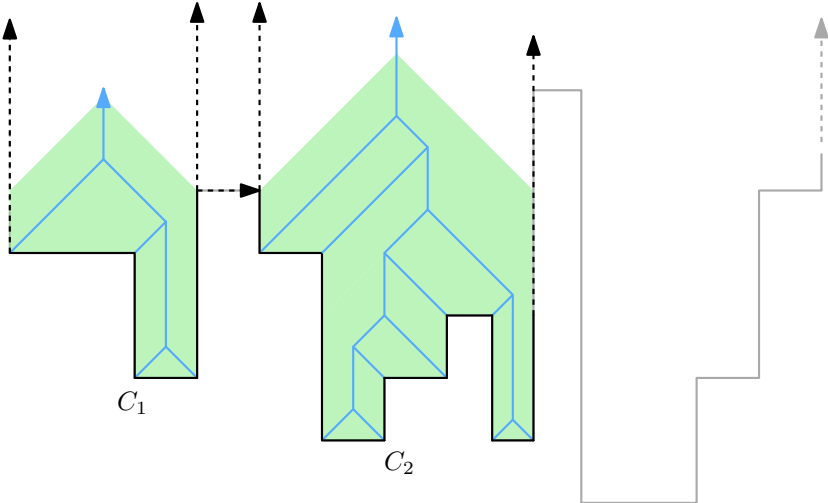
# Straight Skeleton – Algorithm



# Straight Skeleton – Algorithm

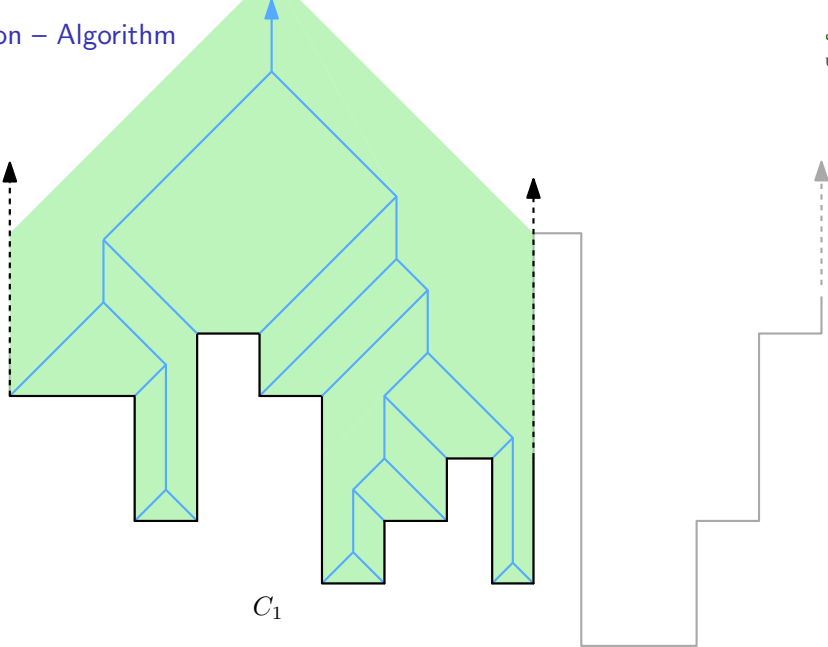


# Straight Skeleton – Algorithm



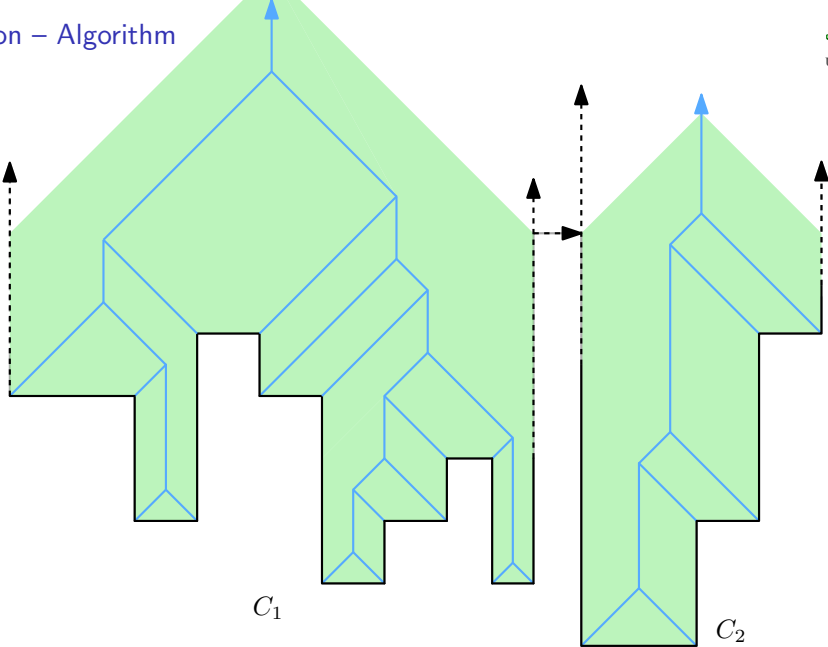


# Straight Skeleton – Algorithm

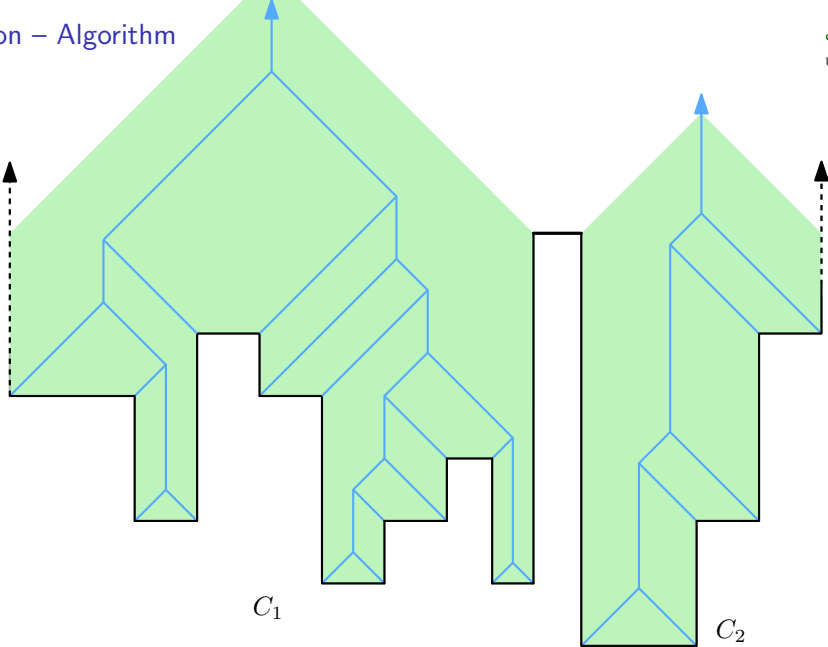


$C_1$

# Straight Skeleton – Algorithm

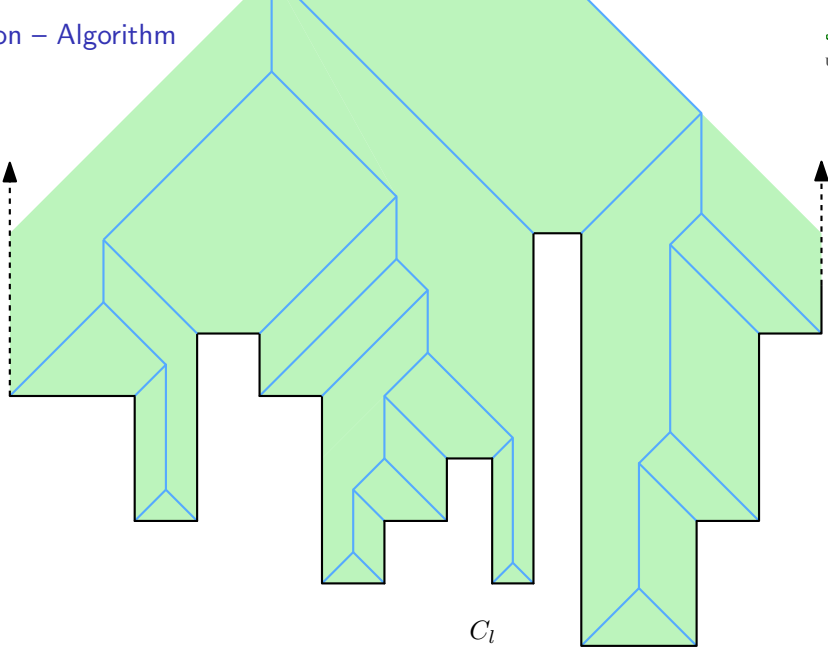


# Straight Skeleton – Algorithm

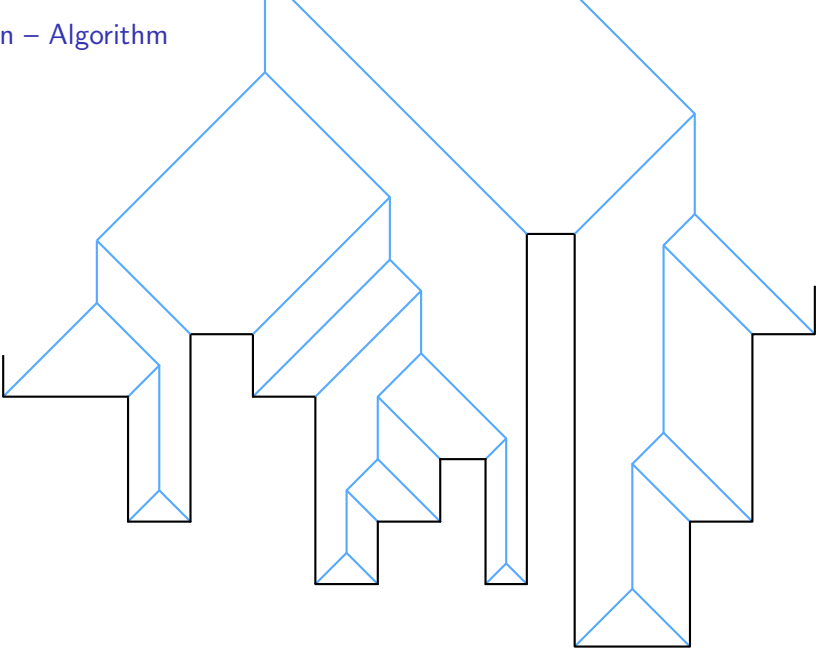




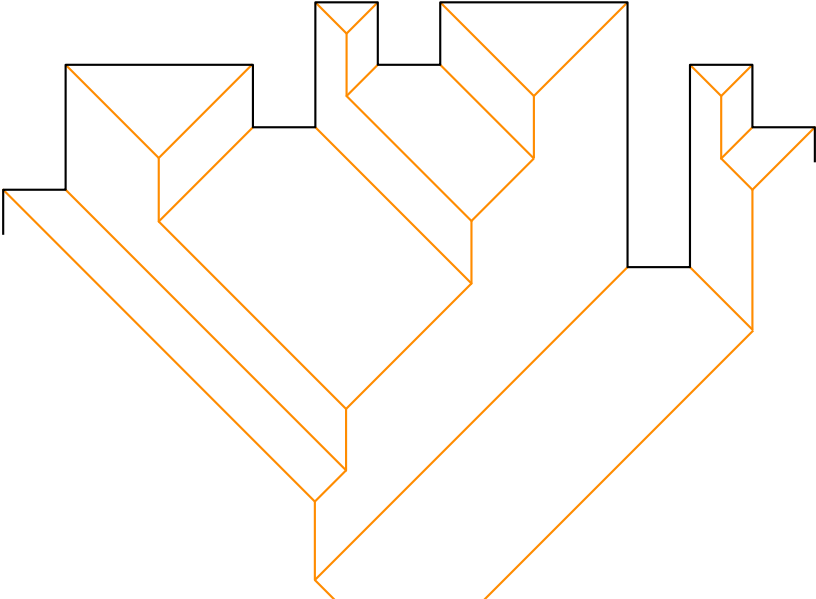
# Straight Skeleton – Algorithm



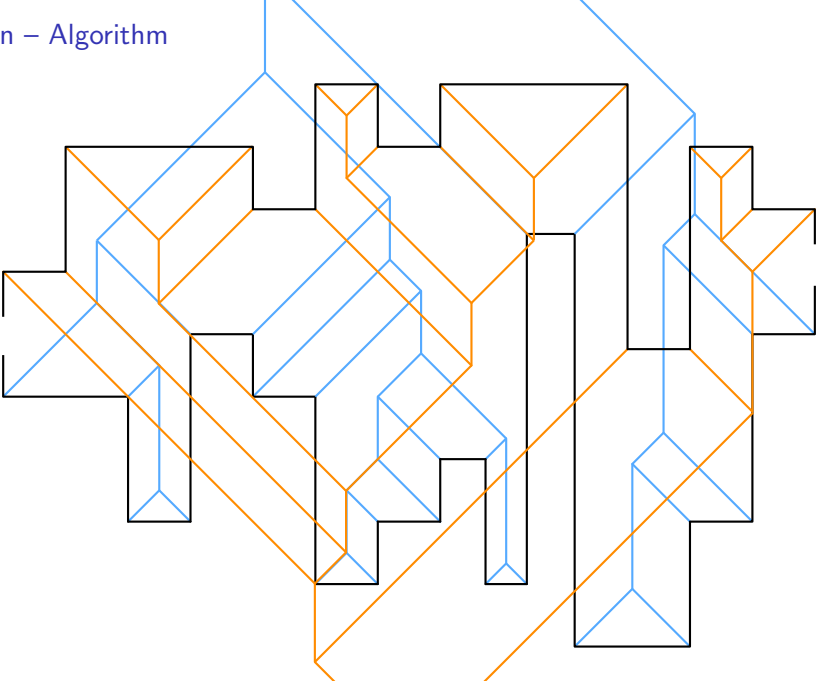
# Straight Skeleton – Algorithm



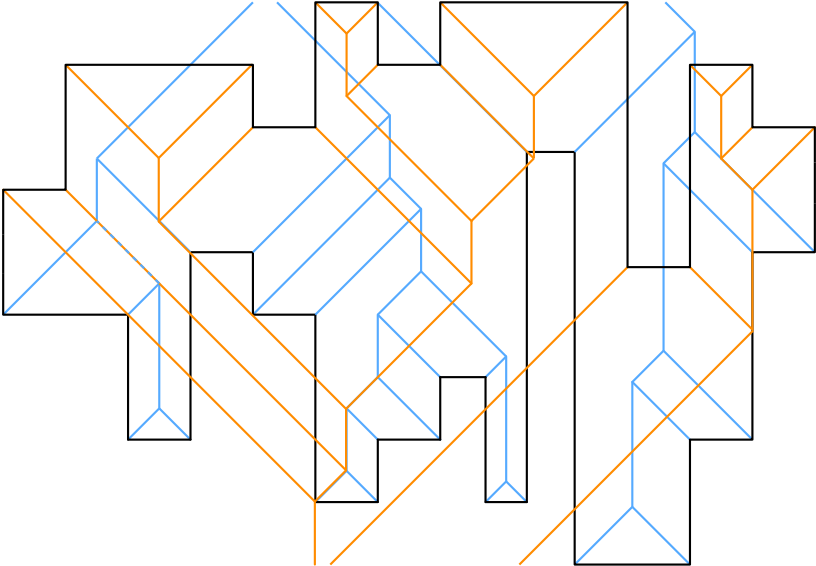
# Straight Skeleton – Algorithm



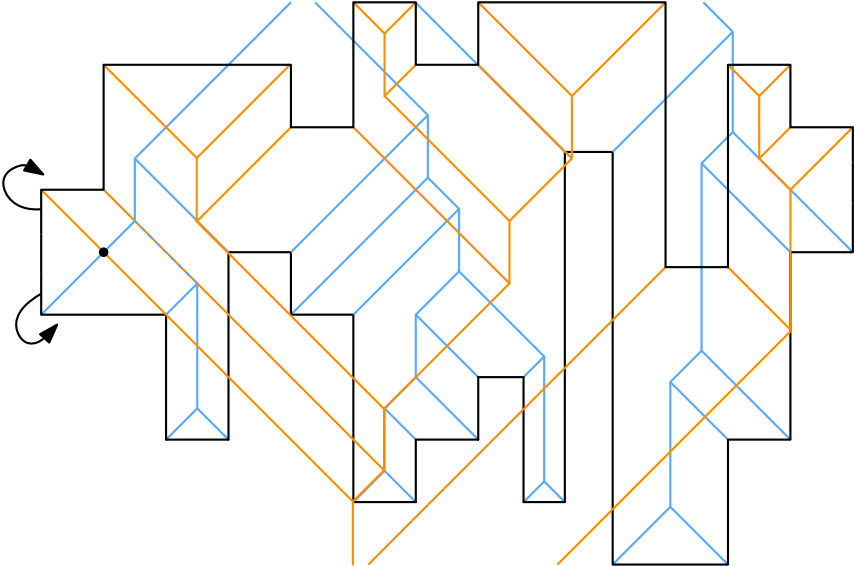
# Straight Skeleton – Algorithm



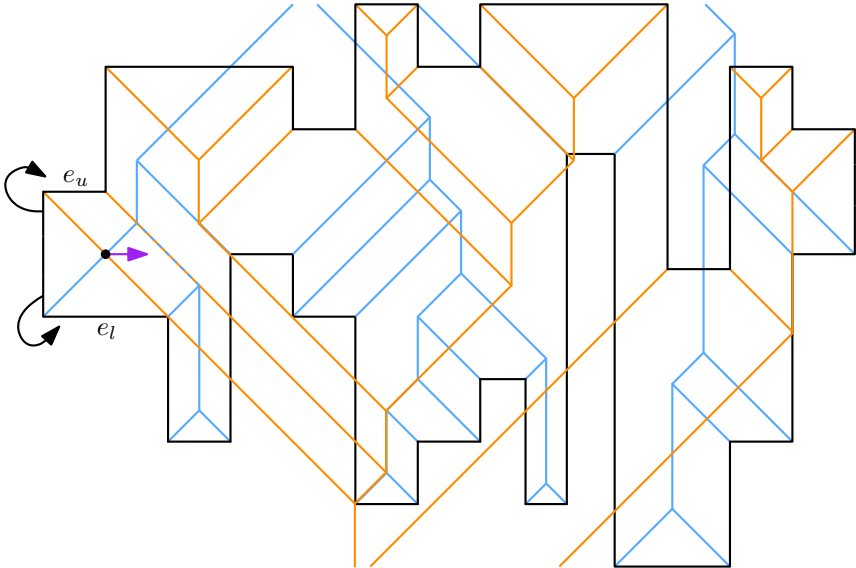
# Straight Skeleton – Algorithm



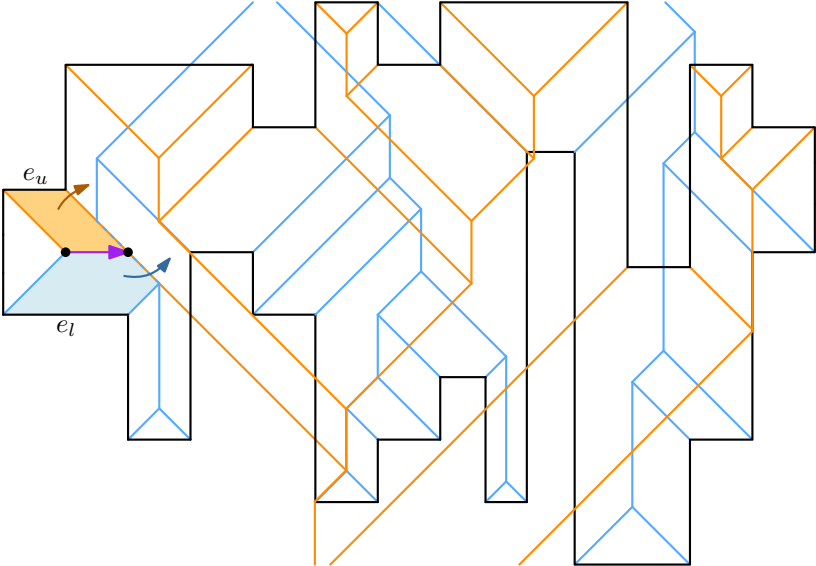
# Straight Skeleton – Algorithm



# Straight Skeleton – Algorithm

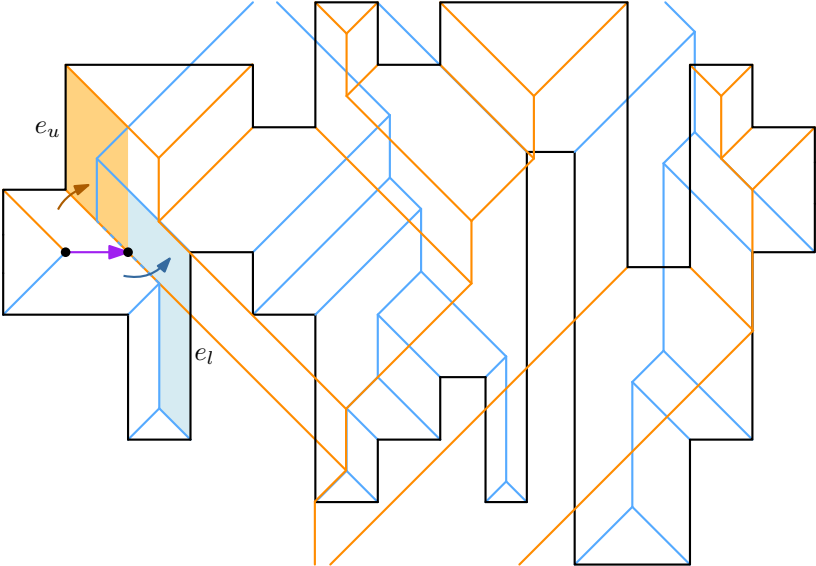


# Straight Skeleton – Algorithm

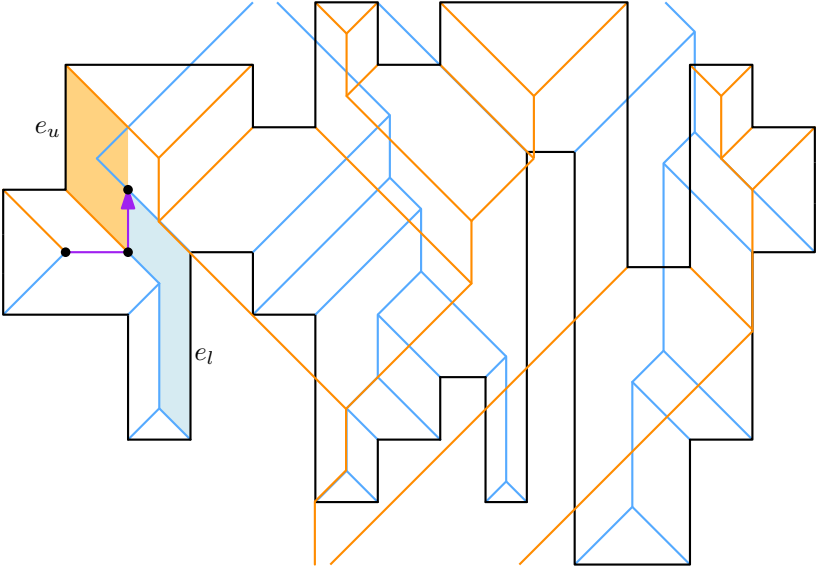




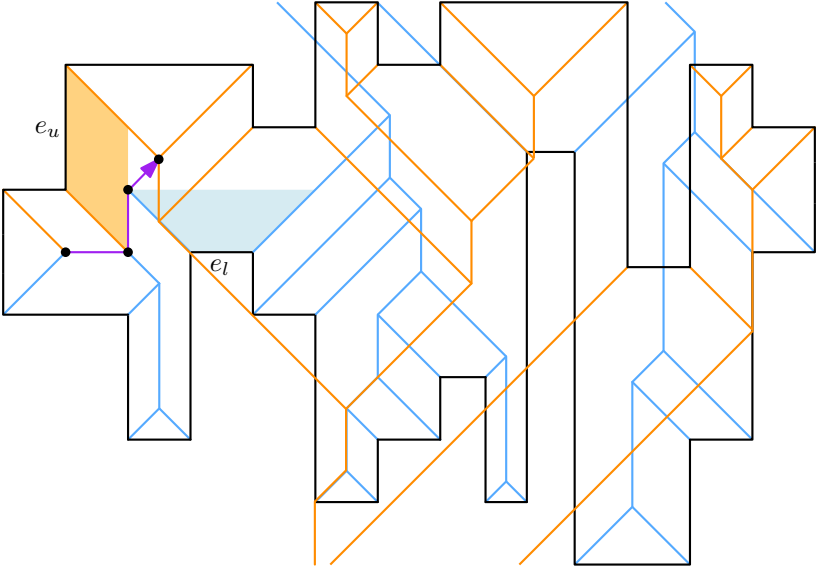
# Straight Skeleton – Algorithm



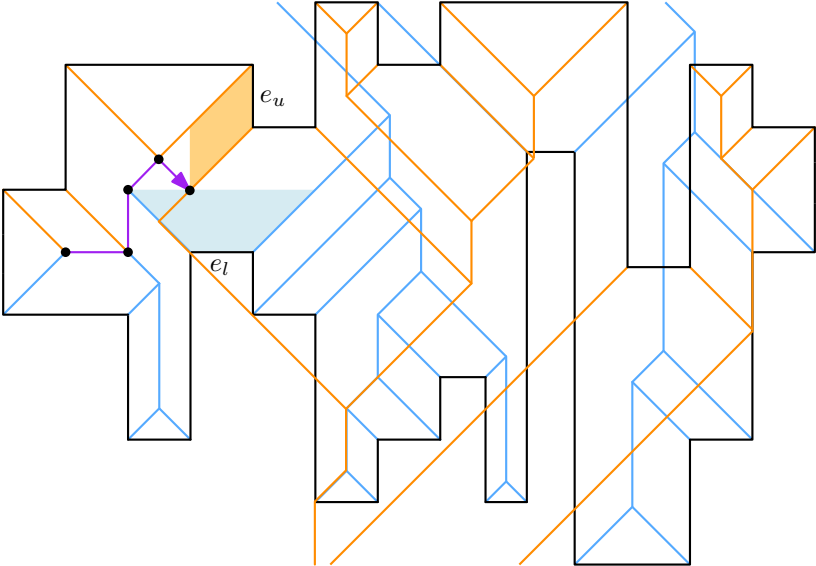
# Straight Skeleton – Algorithm



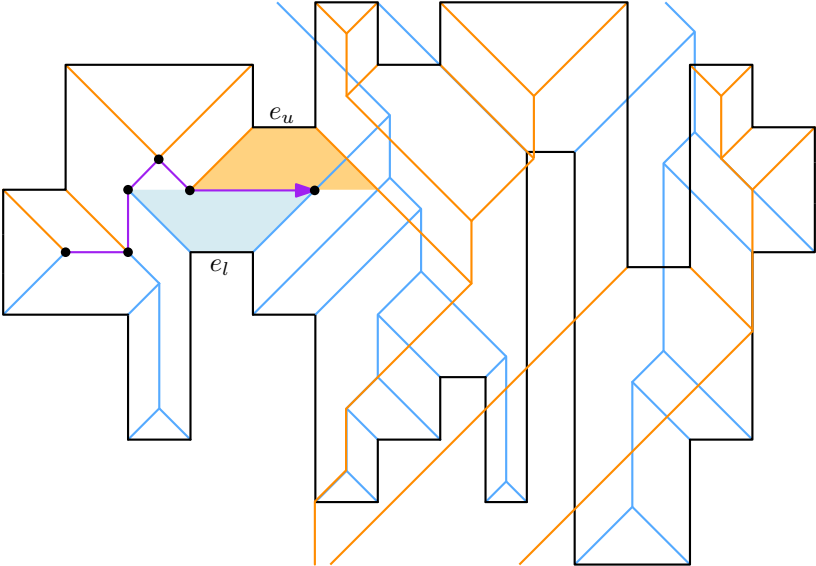
# Straight Skeleton – Algorithm



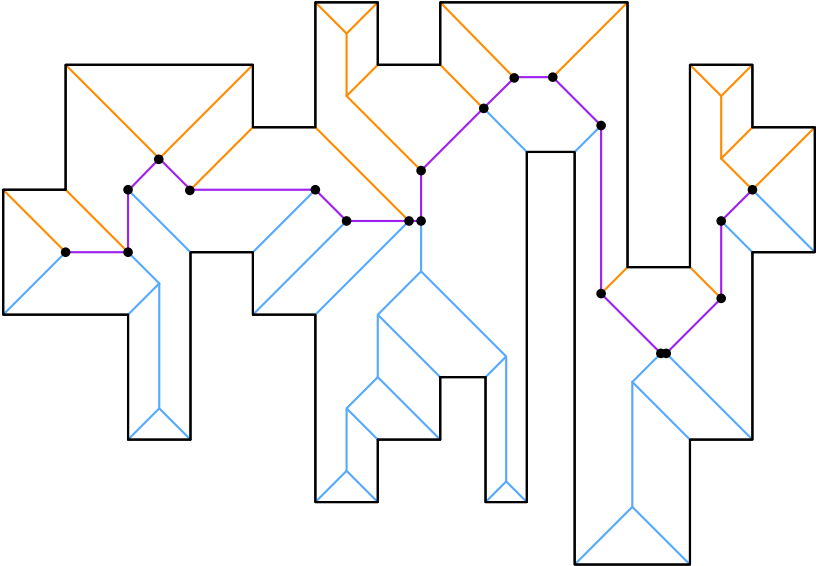
# Straight Skeleton – Algorithm



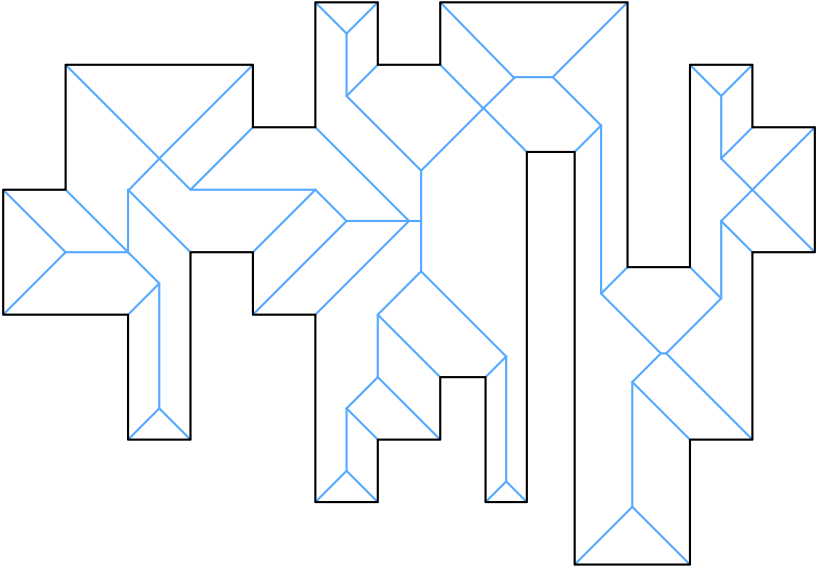
# Straight Skeleton – Algorithm

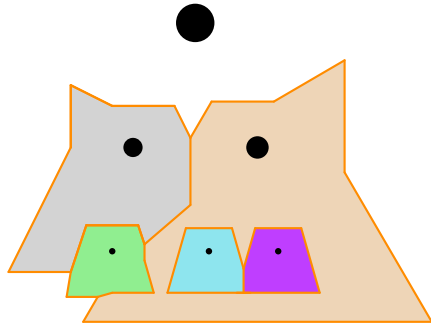


# Straight Skeleton – Algorithm

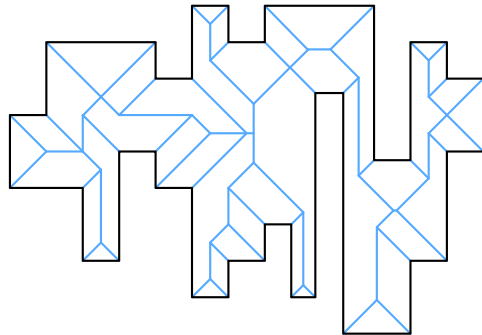


# Straight Skeleton – Algorithm



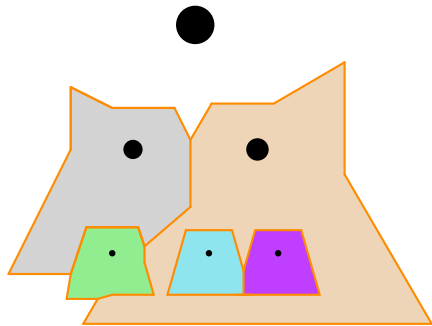


Weighted Voronoi Diagram  $L_\infty$

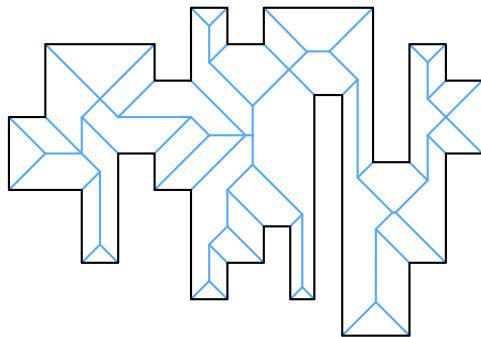


Straight Skeleton





Weighted Voronoi Diagram  $L_\infty$



Straight Skeleton

Questions?

- [1] M. Bock, A. K. Tyagi, J.-U. Kreft, and W. Alt. Generalized voronoi tessellation as a model of two-dimensional cell tissue dynamics. *Bulletin of Mathematical Biology*, 72(7):1696–1731, Oct 2010.
- [2] E. D. Demaine, M. L. Demaine, and A. Lubiw. Folding and One Straight Cut Suffice. In *Proceedings of the 10th Symposium on Discrete Algorithms (SODA 1999)*, pages 891–892.
- [3] D. Eppstein and J. Erickson. Raising Roofs, Crashing Cycles, and Playing Pool: Applications of a Data Structure for Finding Pairwise Interactions. *Discrete & Computational Geometry*, 22(4):569–592, 1999.
- [4] E. Papadopoulou and D. Lee. The  $L_\infty$  Voronoi Diagram of Segments and VLSI Applications. *International Journal of Computational Geometry*, 11(05):503–528, 2001.
- [5] K. Sugihara and Y. Khmelevsky. Roof report from automatically generated 3d building models by straight skeleton computation. In *2018 Annual IEEE International Systems Conference (SysCon)*, pages 1–8, 2018.