Weighted Voronoi Diagrams in the L_∞-Norm

Günther Eder and Martin Held

UNIVERSITÄT SALZBURG
Computational Geometry and Applications Lab

Budapest, June 2018

Voronoi Diagram $\mathcal{V}(S)$

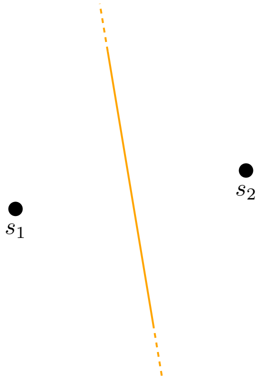
Definition

- Given a set S of *n sites* in $\mathbb{R}^2$.
- Every site s of S defines a *region* $\mathcal{R}(s)$ that contains all points of $\mathbb{R}^2$ closer to s than to any other site.
- The *Voronoi diagram* $\mathcal{V}(S)$ is the union of the boundaries of all n regions.

Voronoi Diagram $\mathcal{V}(S)$

Definition

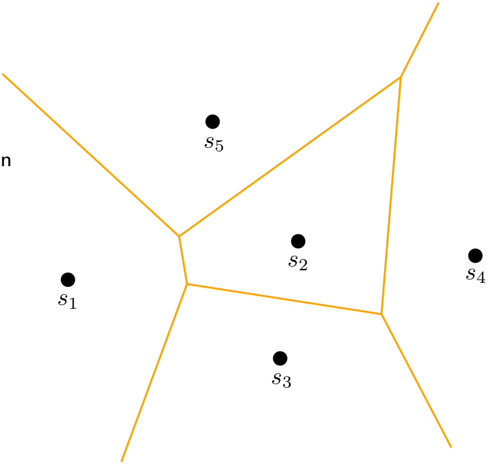
- Given a set S of n *sites* in $\mathbb{R}^2$.
- Every site s of S defines a *region* $\mathcal{R}(s)$ that contains all points of $\mathbb{R}^2$ closer to s than to any other site.
- The *Voronoi diagram* $\mathcal{V}(S)$ is the union of the boundaries of all n regions.



Voronoi Diagram $\mathcal{V}(S)$

Definition

- Given a set S of n *sites* in $\mathbb{R}^2$.
- Every site s of S defines a *region* $\mathcal{R}(s)$ that contains all points of $\mathbb{R}^2$ closer to s than to any other site.
- The *Voronoi diagram* $\mathcal{V}(S)$ is the union of the boundaries of all n regions.



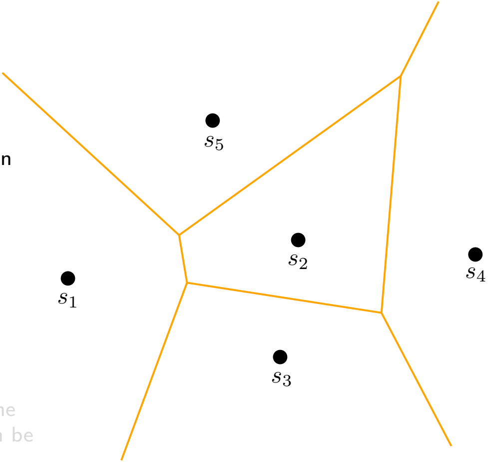
Voronoi Diagram $\mathcal{V}(S)$

Definition

- Given a set S of n *sites* in $\mathbb{R}^2$.
- Every site s of S defines a *region* $\mathcal{R}(s)$ that contains all points of $\mathbb{R}^2$ closer to s than to any other site.
- The *Voronoi diagram* $\mathcal{V}(S)$ is the union of the boundaries of all n regions.

Computation

- Can be computed in $\mathcal{O}(n \log n)$ time and linear space.
- Fortune's sweep line algorithm [2].
- Papadopoulou and Lee[3] show that the Voronoi diagram in the L_∞-metric can be computed using a sweep line as well.



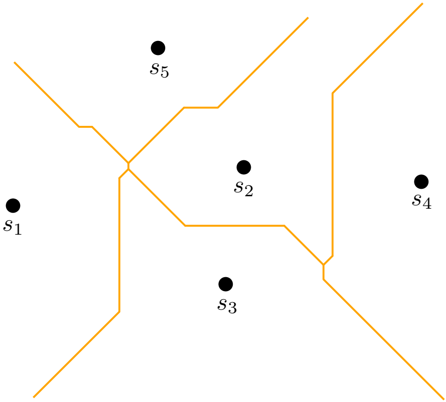
Voronoi Diagram $\mathcal{V}(S)$

Definition

- Given a set S of n *sites* in $\mathbb{R}^2$.
- Every site s of S defines a *region* $\mathcal{R}(s)$ that contains all points of $\mathbb{R}^2$ closer to s than to any other site.
- The *Voronoi diagram* $\mathcal{V}(S)$ is the union of the boundaries of all n regions.

Computation

- Can be computed in $\mathcal{O}(n \log n)$ time and linear space.
- Fortune's sweep line algorithm [2].
- Papadopoulou and Lee[3] show that the Voronoi diagram in the L_∞-metric can be computed using a sweep line as well.



Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w \colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Related Work

- Aurenhammer and Edelsbrunner[1] introduce a worst case optimal approach to compute the weighted Voronoi diagram for L_2 in $\mathcal{O}(n^2)$ time and space.
  - The bisector between two weighted sites in L_2 forms a circle.
  - Spherical inversion.
  - Half-space intersection.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Related Work

- Aurenhammer and Edelsbrunner[1] introduce a worst case optimal approach to compute the weighted Voronoi diagram for L_2 in $\mathcal{O}(n^2)$ time and space.
  - The bisector between two weighted sites in L_2 forms a circle.
  - Spherical inversion.
  - Half-space intersection.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Related Work

- Aurenhammer and Edelsbrunner[1] introduce a worst case optimal approach to compute the weighted Voronoi diagram for L_2 in $\mathcal{O}(n^2)$ time and space.
  - The bisector between two weighted sites in L_2 forms a circle.
  - Spherical inversion.
  - Half-space intersection.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Related Work

- Aurenhammer and Edelsbrunner[1] introduce a worst case optimal approach to compute the weighted Voronoi diagram for L_2 in $\mathcal{O}(n^2)$ time and space.
  - The bisector between two weighted sites in L_2 forms a circle.
  - Spherical inversion.
  - Half-space intersection.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Related Work

- Aurenhammer and Edelsbrunner[1] introduce a worst case optimal approach to compute the weighted Voronoi diagram for L_2 in $\mathcal{O}(n^2)$ time and space.
  - The bisector between two weighted sites in L_2 forms a circle.
  - Spherical inversion.
  - Half-space intersection.

Our Contribution

- The weighted Voronoi diagram in the L_∞-metric, $\mathcal{V}^\infty(S)$, has also a worst case $\Theta(n^2)$ combinatorial complexity.
- Incremental construction approach to construct $\mathcal{V}^\infty(S)$ in $\mathcal{O}(n^2 \log n)$ time.

Multiplicatively Weighted Voronoi Diagrams

Definition

- Let $S := \{s_1, \ldots, s_n\}$ a set of n weighted sites in $\mathbb{R}^2$ and let $w\colon S \to \mathbb{R}^+$ a weight function.
- The region $\mathcal{R}(s)$ contains all points of $\mathbb{R}^2$ closer to s than to any other site of S measured by $d_w(p, s) := {}^{d(p,s)}/_{w(s)}$, where $d(p, s)$ provides the distance in the respective metric.
- $\mathcal{V}(S) := \bigcup_{0<i\leq n} \partial\mathcal{R}(s_i)$, where ∂ denotes the boundary.

Related Work

- Aurenhammer and Edelsbrunner[1] introduce a worst case optimal approach to compute the weighted Voronoi diagram for L_2 in $\mathcal{O}(n^2)$ time and space.
  - The bisector between two weighted sites in L_2 forms a circle.
  - Spherical inversion.
  - Half-space intersection.

Our Contribution

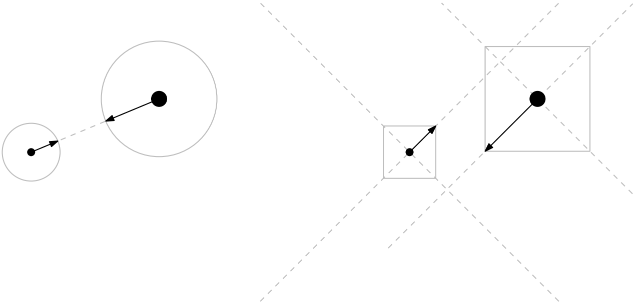
- The weighted Voronoi diagram in the L_∞-metric, $\mathcal{V}^\infty(S)$, has also a worst case $\Theta(n^2)$ combinatorial complexity.
- Incremental construction approach to construct $\mathcal{V}^\infty(S)$ in $\mathcal{O}(n^2 \log n)$ time.

Bisectors

Two weighted sites in the plane and their bisector in the L_2-metric (left) and the L_∞-metric (right).

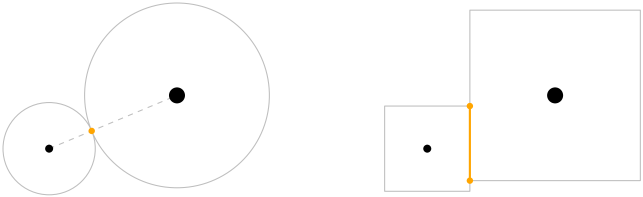
Bisectors

Two weighted sites in the plane and their bisector in the L_2-metric (left) and the L_∞-metric (right).



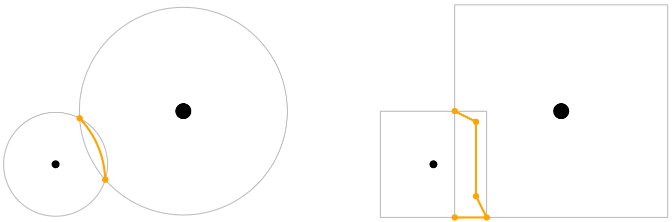
Bisectors

Two weighted sites in the plane and their bisector in the L_2-metric (left) and the L_∞-metric (right).



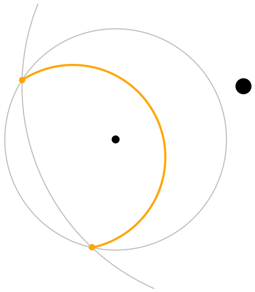
Bisectors

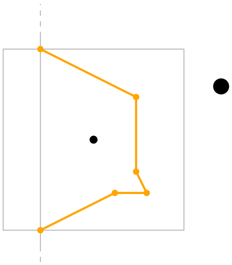
Two weighted sites in the plane and their bisector in the L_2-metric (left) and the L_∞-metric (right).



Bisectors

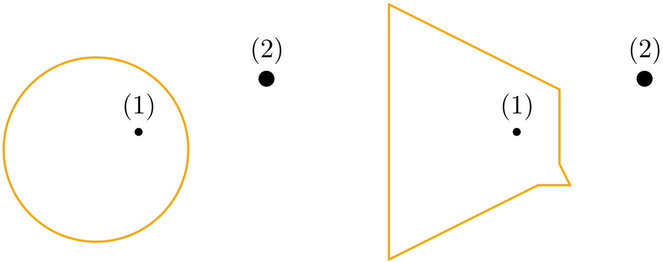
Two weighted sites in the plane and their bisector in the L_2-metric (left) and the L_∞-metric (right).





Bisectors

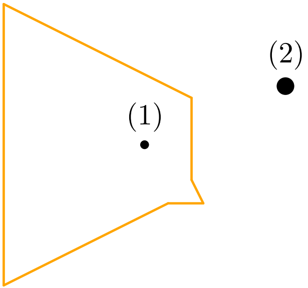
Two weighted sites in the plane and their bisector in the L_2-metric (left) and the L_∞-metric (right).



Bisectors

Two weighted sites in the plane and their bisector in the L_∞-metric .

$\mathcal{V}^\infty(S)$ forms a PSLG$^\infty$.



Bisectors

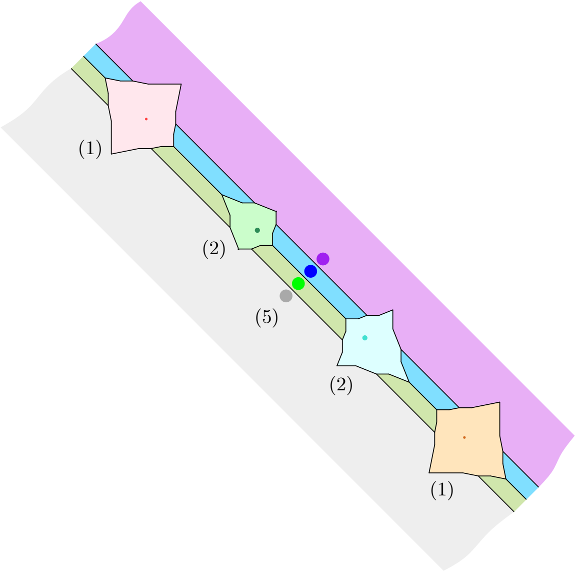
Two weighted sites in the plane and their bisector in the L_∞-metric .

$\mathcal{V}^\infty(S)$ forms a PSLG$^\infty$.

Embedding an axis aligned pyramid on each site. The lower envelope of these pyramids, projected to the plane, forms the bisector of the sites.

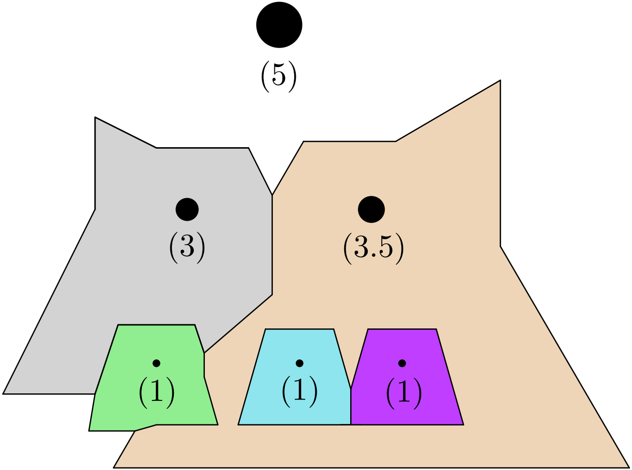
Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

$\Omega(n^2)$ is established by worst case example.



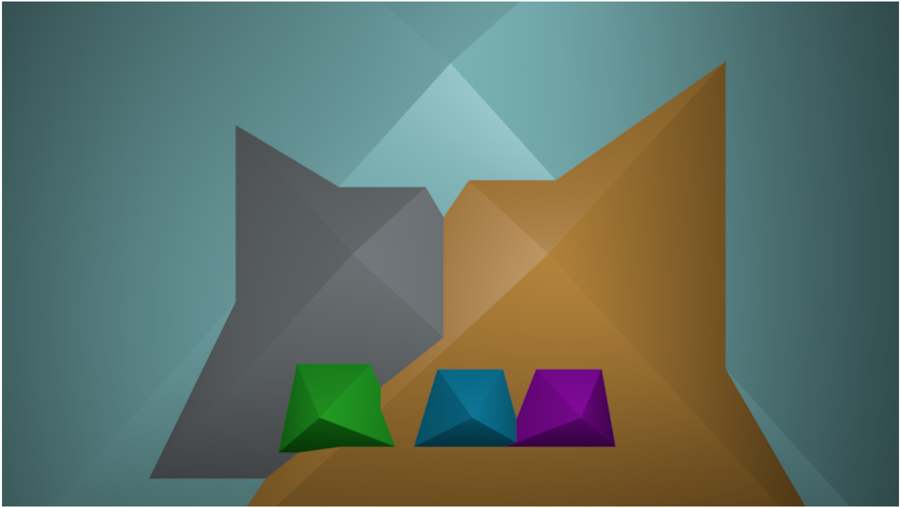
Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

Place an upside-down pyramid p on every site s. The dihedral angle of p is in respect to $w(s)$.

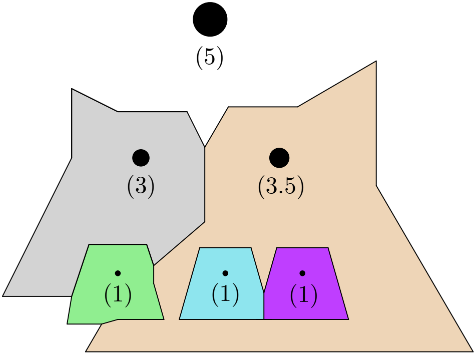


Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

Place an upside-down pyramid p on every site s. The dihedral angle of p is in respect to $w(s)$. Mapping $\mathcal{V}^{\infty}(S)$ onto the set of pyramids. The projection lies on the lower envelop of the pyramids.

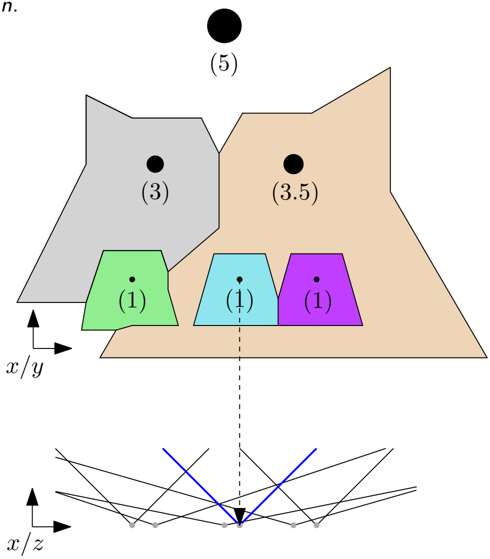


Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$



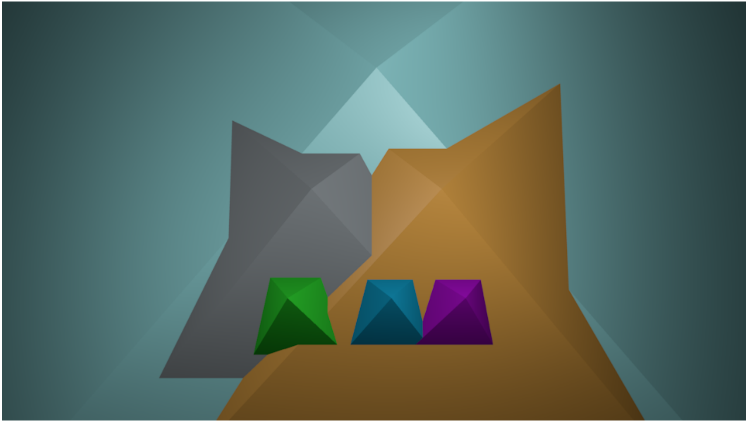
Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

Let $S_k := (s_1, \ldots, s_k)$ be the k sites of S ordered by weight such that $w(s_i) > w(s_{i+1})$, for $0 < i < k \leq n$.



Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

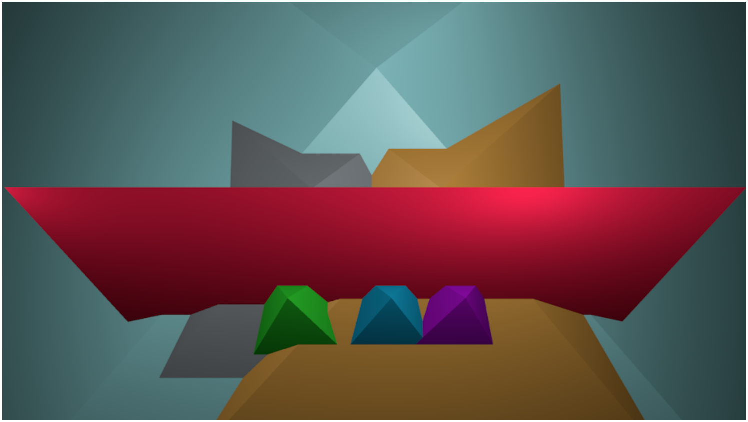
Let $S_k := (s_1, \ldots, s_k)$ be the k sites of S ordered by weight such that $w(s_i) > w(s_{i+1})$, for $0 < i < k \leq n$.



Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

Let $S_k := (s_1, \ldots, s_k)$ be the k sites of S ordered by weight such that $w(s_i) > w(s_{i+1})$, for $0 < i < k \leq n$.

The intersection of a plane (red) with the lower envelope is of size $\mathcal{O}(n)$.



Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

Let $S_k := (s_1, \ldots, s_k)$ be the k sites of S ordered by weight such that $w(s_i) > w(s_{i+1})$, for $0 < i < k \leq n$.

The intersection of a plane (red) with the lower envelope is of size $\mathcal{O}(n)$.

$\mathcal{V}^{\infty}(S)$ has at most $\mathcal{O}(n^2)$ faces, edges, and vertices

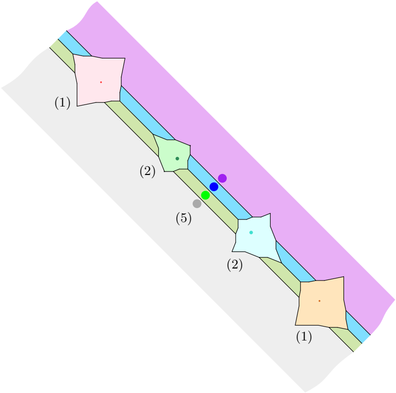
Combinatorial Complexity of $\mathcal{V}^{\infty}(S)$

Let $S_k := (s_1, \ldots, s_k)$ be the k sites of S ordered by weight such that $w(s_i) > w(s_{i+1})$, for $0 < i < k \leq n$.

The intersection of a plane (red) with the lower envelope is of size $\mathcal{O}(n)$.

$\mathcal{V}^{\infty}(S)$ has at most $\mathcal{O}(n^2)$ faces, edges, and vertices

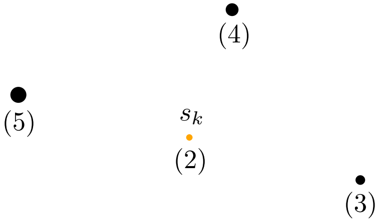
$\mathcal{V}^{\infty}(S)$ has a $\Theta(n^2)$ combinatorial complexity in the worst case.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

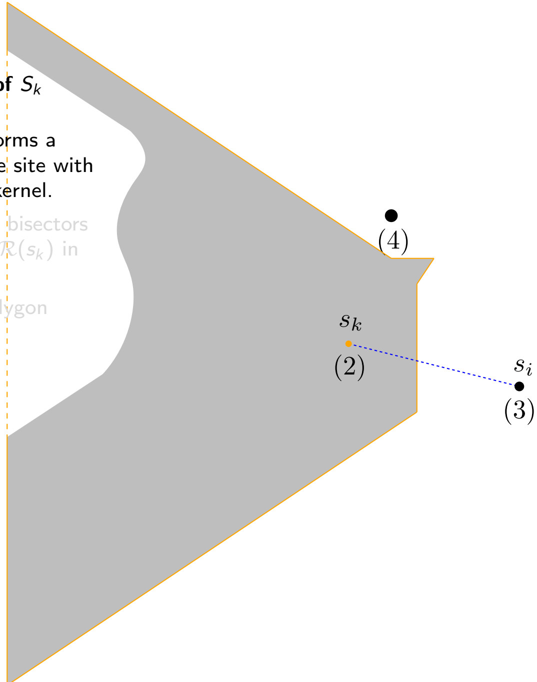
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

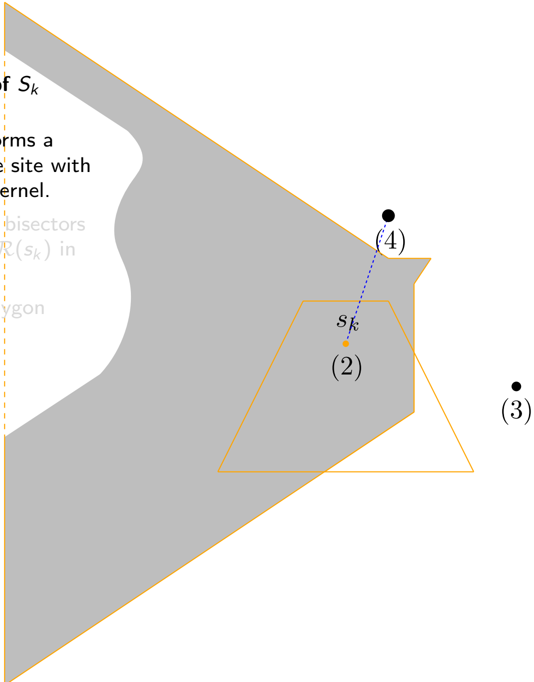
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

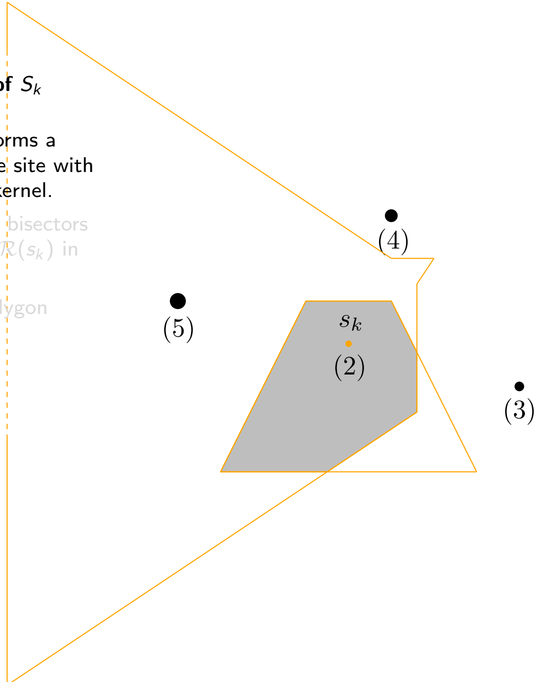
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

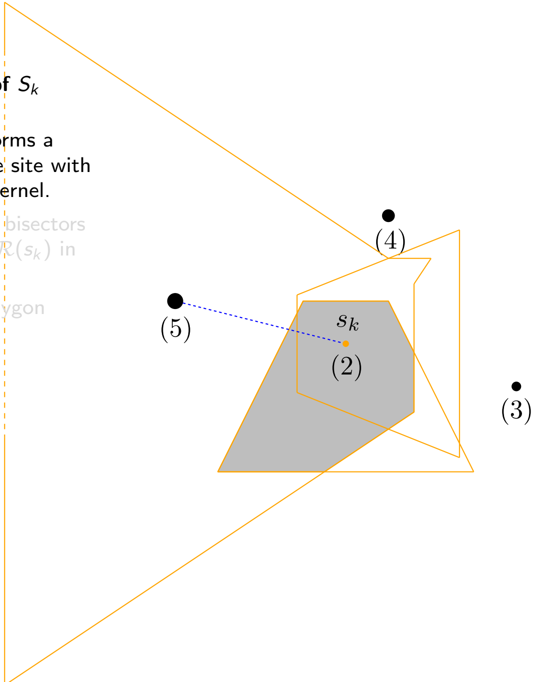
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

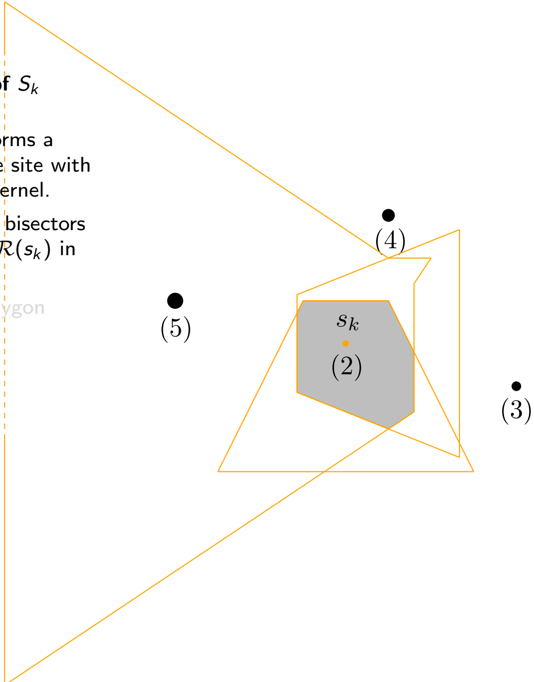
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

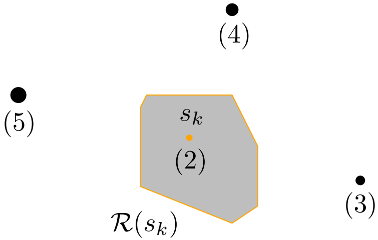
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

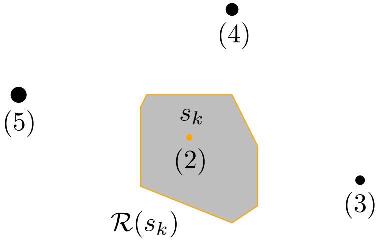
- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



$\mathcal{R}(s_k)$ is Star-Shaped

We construct $\mathcal{R}(s_k)$ for site s_k of S_k

- Bisector between sites s_k, s_i forms a star-shaped polygon where the site with smaller weight resides in the kernel.
- Intersecting the closure of the bisectors of s_k with $s_1, \ldots, s_{k-1}$ forms $\mathcal{R}(s_k)$ in respect to S_k.
- $\mathcal{R}(s_k)$ forms a star-shaped polygon with s_k in its kernel.
- $\mathcal{R}(s_k)$ is linear in size.



Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^{\infty}(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^{\infty}(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^{\infty}(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^{\infty}(S_{k-1})$ inside $\mathcal{R}(s_k)$

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
    - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
    - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

Complexity

- Compute $\mathcal{R}(s_k)$ using a D&C approach in $\mathcal{O}(n \log n)$ time.
- Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$ in $\mathcal{O}(n \log n)$ time.
- Overall we remove at most $\mathcal{O}(n^2)$ edges, where one removal takes $\mathcal{O}(\log n)$ time.
- Therefore, $\mathcal{V}^\infty(S)$ can be constructed in $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(n^2)$ space.

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

Complexity

- Compute $\mathcal{R}(s_k)$ using a D&C approach in $\mathcal{O}(n \log n)$ time.
- Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$ in $\mathcal{O}(n \log n)$ time.
- Overall we remove at most $\mathcal{O}(n^2)$ edges, where one removal takes $\mathcal{O}(\log n)$ time.
- Therefore, $\mathcal{V}^\infty(S)$ can be constructed in $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(n^2)$ space.

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

Complexity

- Compute $\mathcal{R}(s_k)$ using a D&C approach in $\mathcal{O}(n \log n)$ time.
- Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$ in $\mathcal{O}(n \log n)$ time.
- Overall we remove at most $\mathcal{O}(n^2)$ edges, where one removal takes $\mathcal{O}(\log n)$ time.
- Therefore, $\mathcal{V}^\infty(S)$ can be constructed in $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(n^2)$ space.

Incremental Construction

Algorithm

- Compute $\mathcal{R}(s_k)$.
- Compose $\mathcal{V}^\infty(S_k)$ from $\mathcal{R}(s_k)$ and $\mathcal{V}^\infty(S_{k-1})$.
  - Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$.
  - Remove/shorten edges of $\mathcal{V}^\infty(S_{k-1})$ inside $\mathcal{R}(s_k)$

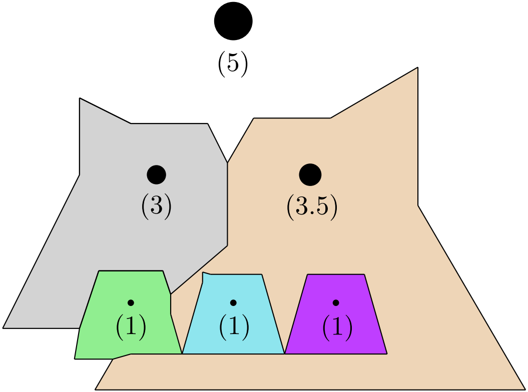
Complexity

- Compute $\mathcal{R}(s_k)$ using a D&C approach in $\mathcal{O}(n \log n)$ time.
- Embed $\mathcal{R}(s_k)$ into $\mathcal{V}^\infty(S_{k-1})$ in $\mathcal{O}(n \log n)$ time.
- Overall we remove at most $\mathcal{O}(n^2)$ edges, where one removal takes $\mathcal{O}(\log n)$ time.
- Therefore, $\mathcal{V}^\infty(S)$ can be constructed in $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(n^2)$ space.

Summary

- Combinatorial complexity in the worst case $\Theta(n^2)$.
- Incremental construction in $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(n^2)$ space.

Questions?



References I

[1] F. Aurenhammer and H. Edelsbrunner. An Optimal Algorithm for Constructing the Weighted Voronoi Diagram in the Plane. *Pattern Recognition*, 17(2):251 – 257, 1984.

[2] S. Fortune. A sweepline algorithm for voronoi diagrams. In *Proceedings of the Second Annual Symposium on Computational Geometry*, SCG '86, pages 313–322, New York, NY, USA, 1986. ACM.

[3] E. Papadopoulou and D. Lee. The L_∞ Voronoi Diagram of Segments and VLSI Applications. *International Journal of Computational Geometry*, 11(05):503–528, 2001.